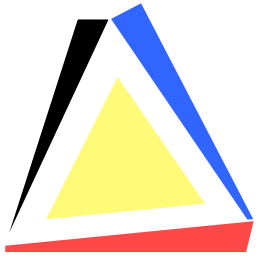

Agent-based Proof Search with Indexed Formulas

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with thanks to Volker Sorge

Saarland University, Saarbrücken, Germany

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What is the talk about?

- CORE: A new basic inference system for ΩMEGA that is based on a (dynamic) rewriting approach
  (developed by Serge Autexier)
- Adapting the suggestion mechanism Ω-ANTS to support proof search in CORE

This is work in progress

CORE: A new basis for ΩMEGA

- Rewriting and focus placement as basic constructs
- Set of rewrite rules dynamically created for each focus
- Overall goal: rewrite problem formalization to $\top$ or $\bot$
- Strong support by simplification; e.g., $\bot \Rightarrow a \vee b$ simplifies to $\top$
- Our motivation: better suited for practical reasoning than conventional calculi
  - treats (propositional) logical aspects implicit; e.g, unwrapping of hypothesis
  - no constraints on the order of quantifier elimination
- System recently applied to interactive proof of $\sqrt{2}$

How does it work ?

Example: $\neg(\neg A \vee \neg B) \wedge (A \Rightarrow (B \Rightarrow C)) \Rightarrow C \vee D$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{C} \vdash \mathbf{C}}}{C \vdash \neg A, \neg B, D, C} \quad \dfrac{\dfrac{\dfrac{}{\mathbf{B} \vdash \mathbf{B}}}{B \vdash \neg A, B, C, D}}{\vdash \neg A, \neg B, C, D, B}}{B \Rightarrow C \vdash \neg A, \neg B, C, D} \quad \dfrac{\dfrac{}{\mathbf{A} \vdash \mathbf{A}}}{A \vdash A, B, \neg B, D, C}}{\vdash A, \neg A, \neg B, C, D}}{A \Rightarrow (B \Rightarrow C) \vdash C, D, (\neg A \vee \neg B)}}{\neg(\neg A \vee \neg B), (A \Rightarrow (B \Rightarrow C)) \vdash C, D}}{\neg(\neg A \vee \neg B) \wedge (A \Rightarrow (B \Rightarrow C)) \vdash C, D}}{\neg(\neg A \vee \neg B) \wedge (A \Rightarrow (B \Rightarrow C)) \vdash C \vee D}}{\vdash \neg(\neg A \vee \neg B) \wedge (A \Rightarrow (B \Rightarrow C)) \Rightarrow C \vee D}$$

Proof of $A \wedge B \wedge (\neg A \vee (B \Rightarrow C)) \Rightarrow C \vee D$ is different!

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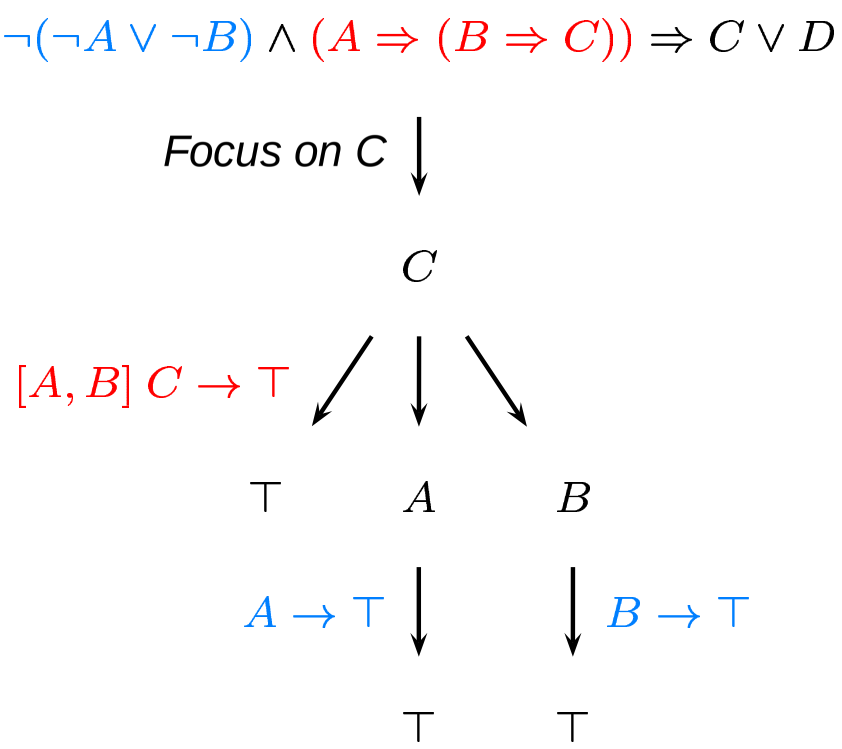
How does it work ?

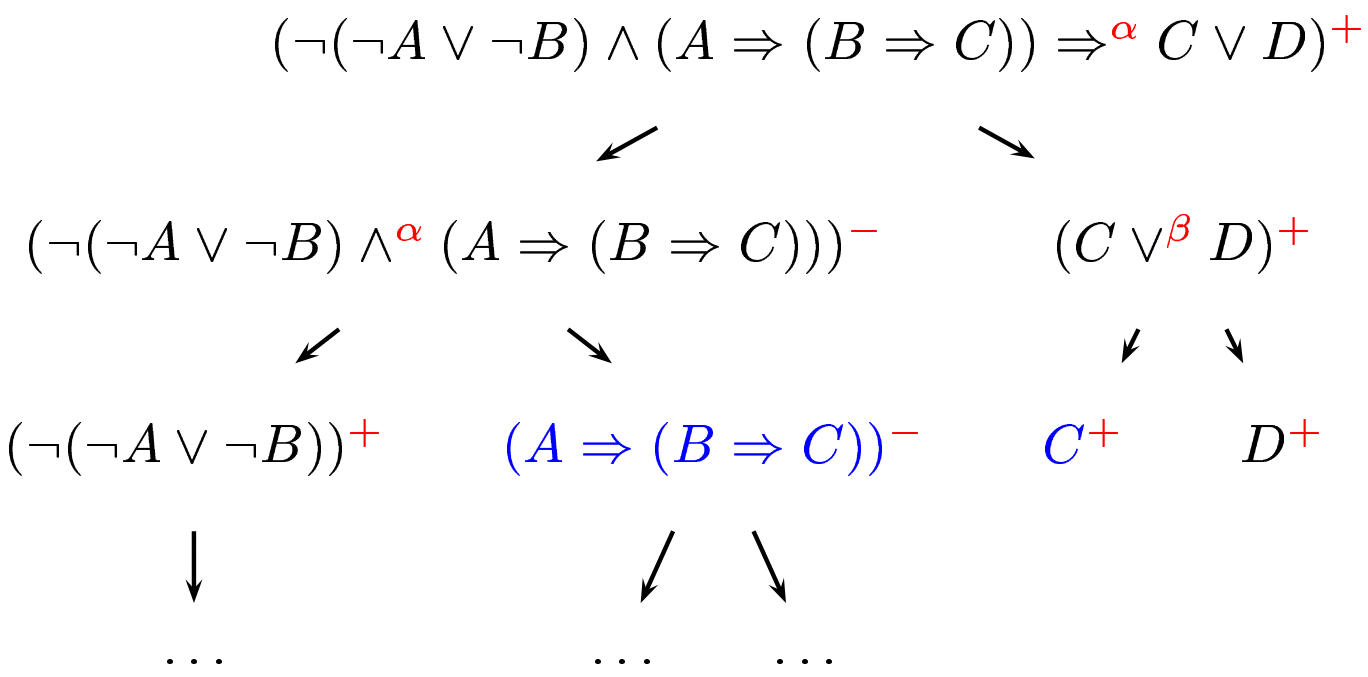
Internal representation: Indexed formula tree



Simplification: $\neg(\neg A \vee \neg B) \wedge (A \Rightarrow (B \Rightarrow C)) \Rightarrow \top \vee D$ simplifies to $\top$

Note: Identical proof for $A \wedge B \wedge (\neg A \vee (B \Rightarrow C)) \Rightarrow C \vee D$.

Indexed formula tree



α-related subformulas belong to the same context r rewriting

Properties of CORE

- New system well suited for interaction with humans (but take care with focus)
- Search Space:
  - Challenge: **choice of focus**
  - Rules of the form $[\Phi]\ u \to v_1 \ldots v_n$ are generated **dynamically**
  - Focus choice and rule selection are interdependent
  - Backtracking
- Advantages for proof search:
  - **big steps** rather than reasoning about logical details
  - **more information is available** in each proof state

Proof Search

To support interactive and automated proof search we want a mechanism that:

- makes suggestions on how to place focus
- computes instances of applicable rewrite rules
- computes instances of additional tactics and methods
- filters and heuristically orders the instances of applicable rules

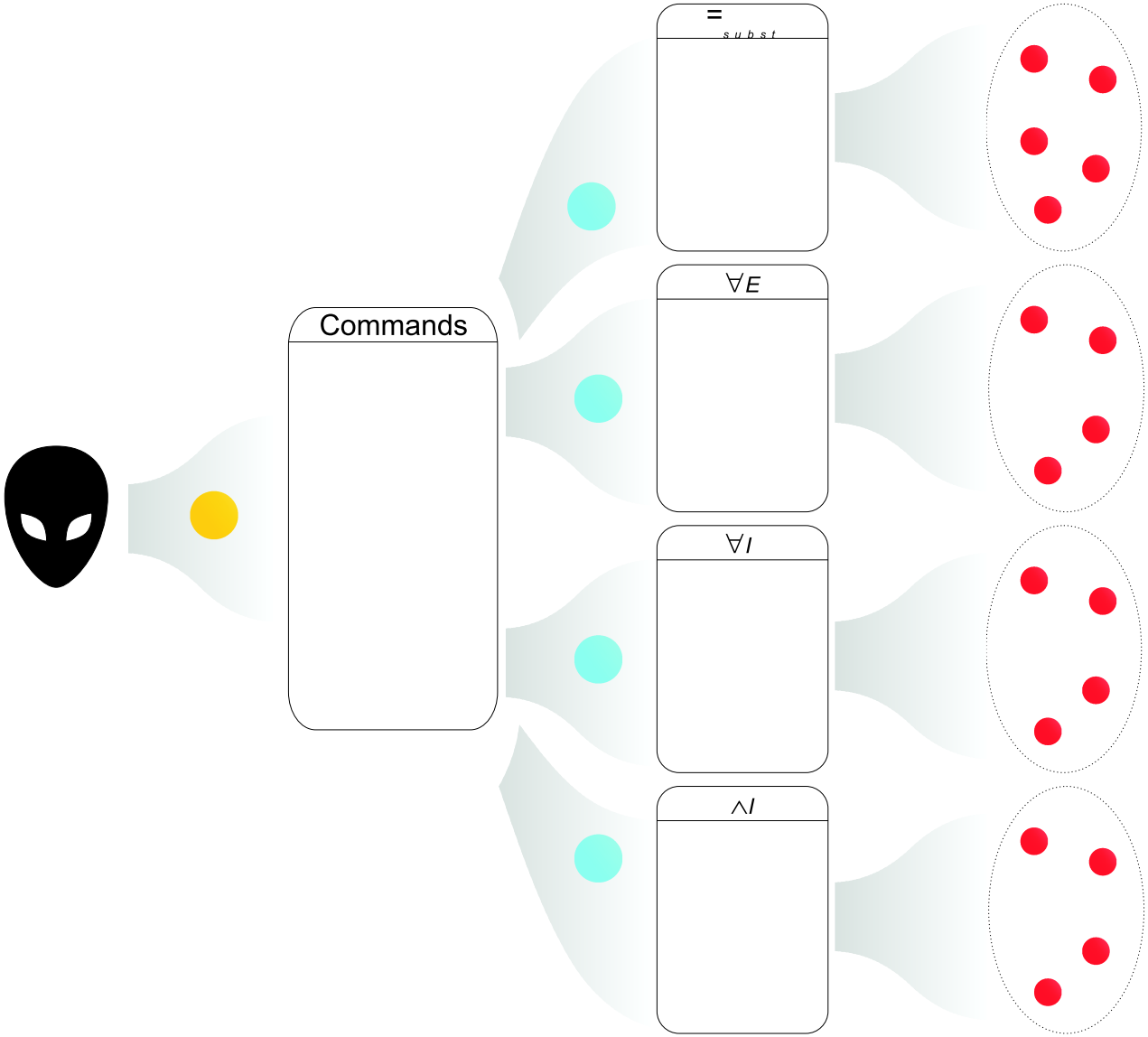
$\Rightarrow$Adapt and employ sugggestion mechanism Ω-ANTS

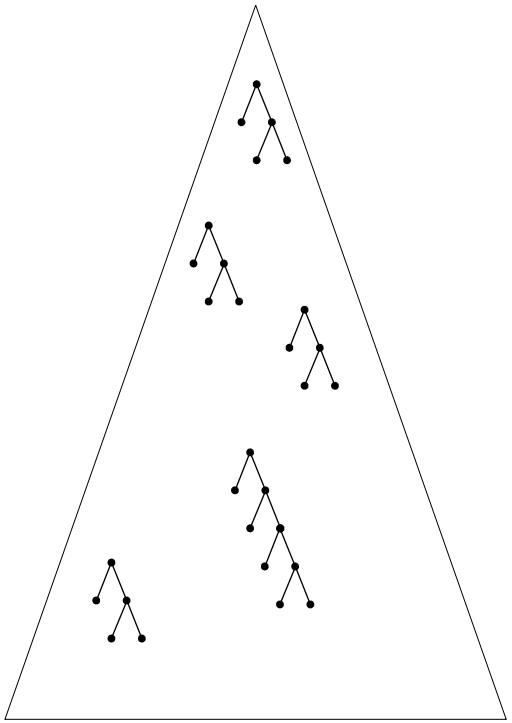
Ω-ANTS

- Key idea:

$$\frac{p_1 \ldots p_m}{c} \; name(t_1, \ldots, t_n)$$

  - Distributed search for instantiations of $p_1, \ldots, p_m, c$ in partial proof; distributed computation of instantiations for $t_1, \ldots, t_m$
  - Communication (about dependencies between parameters) via blackboards
- Certain patterns of instantiations are considered to be applicable
- Heuristically order applicable rules





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Ressourcenadaptive Kognitive Prozesse

Approach for Rewrite Rules

- Rewriting with dynamically generated rewrite rules instead of ND
- Needed: suggestion of applicable rewrite rule instances for current focus
- How to search for applicable instances in formula tree?
  - Model all (dynamic) rewrite rules as agents in Ω-ANTS:
    - dynamic creation of agents for each rule
    + distribution of search for instances
  - Model one generic rewrite agent that handles all rules sequentially:
    + no dynamic creation of agents
    - no distribution of complex instantiation criteria (HO-unification)
- Proposal is to split work
  - one agent that employs FO-unification
  - one or several working with HO-unification

Approach for Tactics/Methods

Static tactics/methods still specified and modeled as before:

$$\frac{p_1, p_2, p_3, p_4, p_5 \ldots, p_n}{c}\ name(t_1, \ldots, t_m)$$

Applicable *partial argument instantiations* have to be cast into rewrite rules for application in the system

- Use Ω-ANTS as it is
- Agents have to search for instantiations in indexed formula tree
- Transform each applicable instantiation pattern in rewrite rule:
  If c, p_1, p_3 is the set of parameters for which instantiations were found

$$c \rightarrow p_2 \wedge p_4 \wedge \ldots \wedge p_n$$

Approach for Tactics/Methods

$$\frac{p_1 : A \Rightarrow B \quad p_2 : B \Rightarrow C}{c : A \Rightarrow C}\ ModusBarbara$$

Example: $(P(a) \Rightarrow Q(a)) \wedge A(f(g)) \wedge R(y) \Rightarrow (P(a) \Rightarrow R(a))$

$$\frac{p_1 : P(a) \Rightarrow Q(a) \quad p_2 : \emptyset}{c : P(a) \Rightarrow R(a)}\ ModusBarbara$$

Focusing on $P(a) \Rightarrow R(a)$ should yield the following rewrite rules:

$$(P(a) \Rightarrow R(a)) \rightarrow (Q(a) \Rightarrow R(a))$$

$$[Q(a) \Rightarrow R(a)] \ \ P(a) \rightarrow R(a)$$

Approach for Focus Placement

Focus Agent:

- Suggests to consider certain foci and excludes other foci from the search
  - Criteria from theoretical investigations for heuristics/strategies
  - Criteria of human mathematicians
- Problem: focus placement can easily become to complicate for humans
  - user model needed for optimal support ?

Winding up

- Adaptation of Ω-ANTS to support interactive proof search in CORE
  - Difficulty: interplay between focus placement, dynamic generated rewrite rules, tactics/methods, and backtracking
- Current focus of work: Interactive theorem proving
  Possible evaluation:
  Get a novice user (e.g. maths student) to replay his blackboard proof of a mathematical problem such as *irrationnality of* $\sqrt{2}$ in the system as adequately and as comfortably as possible
- Future work: Automation of proof search based on Ω-ANTS and mixed-initiative proof search