



## *System Description*

The OMEGA Group

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SFB 378  
Ressourcenadaptive  
Kognitive Prozesse

# Research in the OMEGA project

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Aim: assistant for the working mathematician

Means: development and integration of heterogenous tools

- reasoning proof planning (PP), agent-based reasoning, ATP
- computation computer algebra
- interaction tactical TP, mixed initiative PP
- proof maintenance proof object, diff. levels of detail
- user interface graphical UI, natural language
- knowledge management mathematical database
- infrastructure network of service systems

OMEGA project :=

collection of integrated heterogeneous research projects linked

via the core OMEGA-system

Multi-Strategy Proof  
Planning

Agent-based  
Reasoning

Interactive Theorem  
Proving

External Support  
Systems

Knowledge-based Proof Planning

[MELISSIEKMANN99],[MELISMEIER00]

- Proof method: domain specific, mathematically motivated, **may encapsulate and employ external systems**
- Control knowledge: heuristic information on how to guide plan search, **may employ external systems**
- Strategy := (set-of-proof-methods, control-knowledge)

Logic & Calculi

Case Studies

Proof Planning with MULTI:

[MELISMEIER00]

- Supports interleaving of different strategies

Graphical User  
Interface

Natural Language  
Proof Presentation

Mathematical  
Knowledge Base

Mathematical  
Software Bus

# Agent-based Reasoning

Multi-Strategy Proof Planning

Agent-based Reasoning

Interactive Theorem Proving

External Support Systems

Agent-based Reasoning . . . [BENZMÜLLERSORGE99][BENZMÜLLERETAL-00]

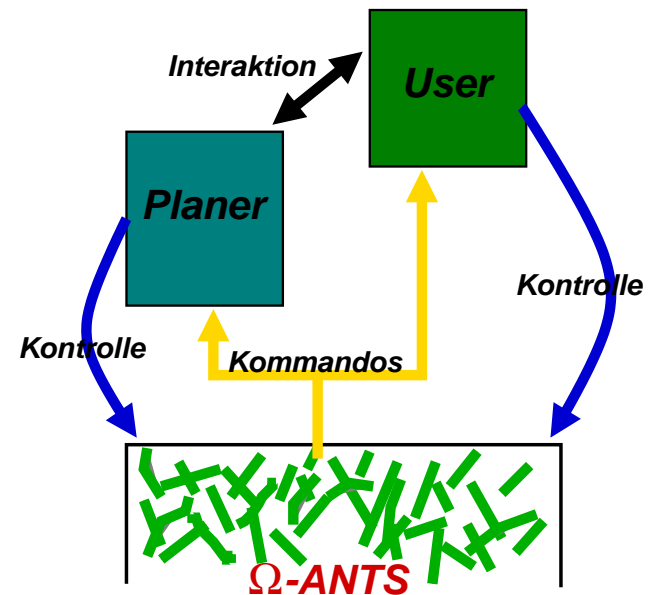
Logic & Calculi

- Idea: model methods, tactics, external systems as pro-active entities

Interactive Theorem Proving

- Suggestion mechanism with anytime character
- Mixed-initiative proof planning

Case Studies



Graphical User Interface

Natural Language Proof Presentation

Mathematical Knowledge Base

Mathematical Software Bus

Multi-Strategy Proof Planning

Agent-based Reasoning

Interactive Theorem Proving

External Support Systems

External Systems ... [BENZMÜLLER ET AL 99], [MELIS ET AL 00], [SORGE 00]

- as subsystems in proof methods or control rules

- as own tactics  $\frac{P_1 \dots P_n}{C}$  *Otter*  $\frac{P}{C}$  *SimplifyWithMaple*

- Soundness: Transformation in proof plan, Expansion, Checking

- Tool Support for Transformation:

- TRAMP: FO refutation proofs [MEIER 00]

- SAPPER: CAS computations [SORGE 00]

Logic & Calculi

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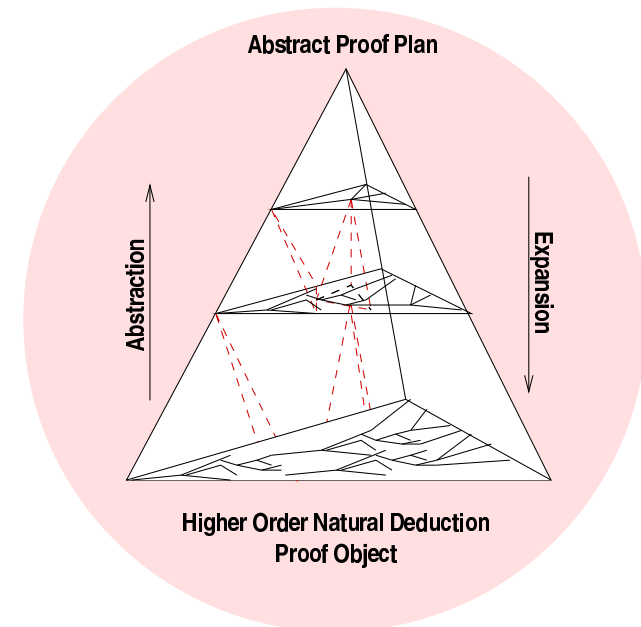
External Support  
Systems

OMEGA proof data structure (PDS)

[CHEIKHR.SORGE00]

Logic & Calculi

- maintains proof plans at **different levels of detail** simultaneously
- base layer: higher order ND variant for simply typed  $\lambda$ -calculus
- abstract level proof plans not necessarily sound: **verification by expansion** to more detailed layer



Case Studies

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# User Interface

Multi-Strategy Proof Planning

Agent-based Reasoning

Interactive Theorem Proving

External Support Systems

Graphical user interface  
*LOUI* [SIEKMANN ET AL 99]

Logic & Calculi

- multi-modal: proof tree, linearized proof, term browser
- hypertext mechanism
- natural language proof presentation

Label	Hypothesis	Term	Method	Premises
L11	L8	$((\text{sqrt } 2) * n) = m$	ANDE*	L8
L12	L8	$\neg(\text{exists-sort } (\lambda dc-270, (\text{comm$	ANDE*	L8
L13	L8 L4 RAT-CRT	$(\text{power } m \ 2) = (2 * (\text{power } n$	BY-COMPUTATIO	L11
L14	L8 L4 RAT-CRT	$\text{evenp } (\text{power } m \ 2)$	DefInE	L15
L15	L8 L4 RAT-CRT	$\text{exists-sort } (\lambda dc-278, ((\text{power$	EXISTSI-SORT	L13 L16
L16	L8 L4 RAT-CRT	$\text{int } (\text{power } n \ 2)$	WELLSORTED	L6
SQUARE-EVEN	SQUARE-EVEN	$\text{forall-sort } (\lambda x, ((\text{evenp } (\text{pow$	THM	
L17	SQUARE-EVEN	$\text{evenp } m$	ASSERT	SQUARE-EVEN L10 L1
L18	SQUARE-EVEN	$\text{exists-sort } (\lambda dc-334, (m = (2$	DefInE	L17
L19	L19	$(\text{int } k) \wedge (m = (2 * k))$	HYP	
L20	SQUARE-EVEN	$\perp$	WEAKEN	L23
L21	L19	$\text{int } k$	ANDE	L13
L22	L19	$m = (2 * k)$	ANDE	L13
L23	L19 SQUARE-EV	$(\text{power } n \ 2) = (2 * (\text{power } k$	BY-COMPUTATIO	L13 L24
L24	L19 SQUARE-EV	$\text{evenp } (\text{power } n \ 2)$	DefInE	L25
L25	L19 SQUARE-EV	$\text{exists-sort } (\lambda dc-344, ((\text{power$	EXISTSI-SORT	L23 L26
L26	L19 SQUARE-EV	$\text{int } (\text{power } k \ 2)$	WELLSORTED	L21
L27	L19 SQUARE-EV	$\text{evenp } n$	ASSERT	SQUARE-EVEN L6 L24
EVEN-COMMON-D	EVEN-COMMON-D	$\text{forall-sort } (\lambda x, (\text{forall-sort$	THM	
L28	EVEN-COMMON-D	$\text{int } 2$	WELLSORTED	
L29	EVEN-COMMON-D	$1$	ASSERT	EVEN-COMMON-DIVISO

**Pretty Term**

```

( $\lambda x, (\text{forall-sort } (\lambda y, ((\text{evenp } x) \wedge (\text{evenp } y)) = (\text{common-divisor } x \ y \ 2))) \text{int}$ )
int
evenp n
int (power k 2)
exists-sort ( $\lambda dc-344, ((\text{power } n \ 2) = (2 * dc-344))) \text{int}$ 
(power m 2) = (2 * (power n 2))
m = (2 * k)
(power n 2) = (2 * (power k 2))

```

Output Message Error Warning Trace

```

:::CSM Creator [2]: Command agent for command NEUT
:::CSM Creator [2]: Defining 3 default agents For
:::CSM Creator [2]: Blackboard for command INVERSE
:::CSM Creator [2]: Command agent for command INVE
:::CSM Creator [2]: Defining 2 default agents For
:::CSM Creator [2]: Blackboard for command APPLY-A
:::CSM Creator [2]: Command agent for command APPL
:::CSM Creator [2]: Defining 2 default agents For
:::CSM Creator [2]: Blackboard for command DEFNDP
:::CSM Creator [2]: Command agent for command DEFN
:::CSM Creator [2]: Defining 1 default agents For
:::CSM Creator [2]: Blackboard for command REFLEX
:::CSM Creator [2]: Command agent for command REFL

```

"Agents now NOT resource adaptive!" Initialized h

Total: 33 Depth: 0 Command: Execute-Theorem-Log Time: 1.56s

Case Studies

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# User Interface

Multi-Strategy Proof Planning

Agent-based Reasoning

Natural language proof presentation with *P.r*ex

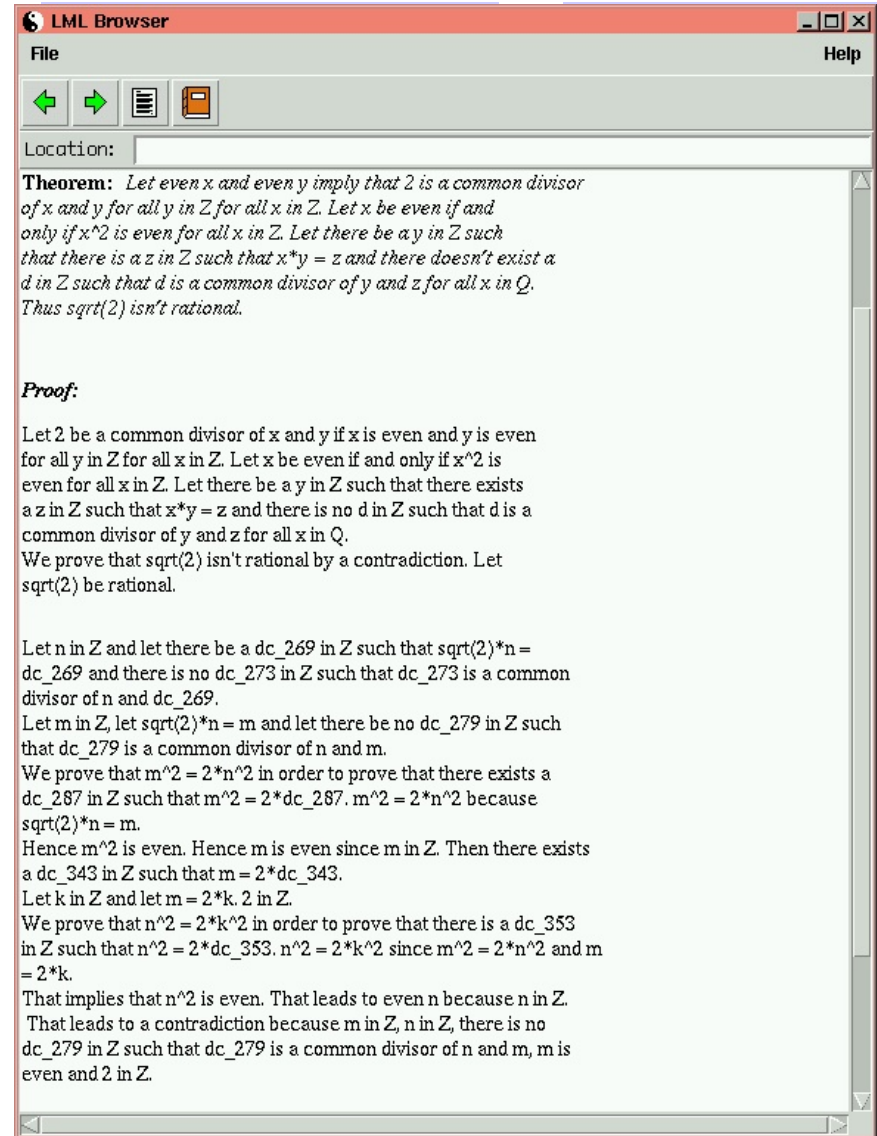
[FIEDLER01]

- interactive and adaptive
- supports different levels of detail

Logic & Calculi

Graphical User Interface

Natural Language Proof Presentation



Support

Case Studies

Cal

Knowledge Base

Software Bus





# Knowledge Management & Infrastructure

## Multi-Strategy Proof Planning

## Agent-based Reasoning

## Interactive Theorem Proving

## External Support Systems

MBASE [FRANKEKOHLHASE00]

- currently: migration from old knowledge base to MBASE
- ideally: MBASE maintains **all** domain specific knowledge

MATHWEB [ZIMMERKOHLHASE02]

- OMEGA environment

=

bunch of MATHWEB tools orchestrated by the OMEGA core system

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$\epsilon - \delta$  proofs

[MELISSIEKMANN99]

- Maple for manipulation of arithmetic expressions
- constraint solver *CoSIE* for instantiation of meta-variables

Classification of residue class structures, e.g.  $(\mathbb{Z}_5, \bar{*})$  [MEIERETAL00]

- which algebr. structure (semi-group, monoid, etc.)?, isomorphic?
- more than 10.000 structures analyzed
- 3 essentially **different kinds of proofs**

Recently: Irrationality of  $\sqrt{2}$ ; interactive and automated with PP

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# Conclusion

**Multi-Strategy Proof  
Planning**

**Agent-based  
Reasoning**

**Interactive Theorem  
Proving**

**External Support  
Systems**

Is OMEGA already a tool for the working mathematician?

- No ... but the perspective is there

What is needed?

- Better integration of tools
- Improvement (research!) of all tools
- Software engineers to turn research products in solid system

Logic & Calculi

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