Critical Agents Supporting Interactive Theorem Proving

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Overview

- Motivation
- A layered blackboard mechanism for suggesting promising proof steps
- Guiding the search:
  - A resource adapted approach
  - A resource adaptive approach
- Applications & Conclusions

Tactical Theorem Proving in ΩMEGA

- Natural deduction (ND) rules
- Tactics are macro steps (expandable in a sequence of ND rule applicationsquence of ND rule applications)
- Proof methods are tactics enriched by a declaratively specified pre- and post-conditions
- Other commands:
  - call external systems like OTTER or LEO
  - verbalise a proof, verify a proof, ...
- This talk addresses: rules, tactics, methods, and external systems

$$\frac{\forall x_{\bullet} A}{[t/x]A}\ \forall_E(t) \qquad \frac{\Phi[x]_p \quad x = y}{\Phi'[y]_p}\ =_{Subst}(p)$$

State of the Art & Research Goals

- Drawback of respective suggestion mechanisms in state of the art theorem proving environments (e.g. ΩMEGA, TPS, HOL):

  pure passive character

- A good suggestion mechanisms instead should
  - autonomously run in the background of the system
  - dynamically present the most promising suggestions to the user
  - react and respond to special queries by the user
  - optimally employ the available resources
  - ...
- Research goal: active and intelligent proof assistant

Example ND-Proof

Problem: $p_{o \to o}\ (a_o \wedge b_o) \Rightarrow (p\ (b \wedge a))$

ND-proof:

C	$()$	$\vdash$	$(p\ (a \wedge b)) \Rightarrow (p\ (b \wedge a))$	OPEN

Example ND-Proof

Problem: $p_{o \to o}\,(a_o \wedge b_o) \Rightarrow (p\,(b \wedge a))$

ND-proof:

L_1	(L_1)	$\vdash$	$(p\,(a \wedge b))$	Hyp
L_2	(L_1)	$\vdash$	$(p\,(b \wedge a))$	OPEN
C	$()$	$\vdash$	$(p\,(a \wedge b)) \Rightarrow (p\,(b \wedge a))$	$\Rightarrow_I$: (L_2)

Example ND-Proof

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ND-proof:

L_1	(L_1)	$\vdash$	$(p\ (a \wedge b))$	Hyp
L_3	(L_1)	$\vdash$	$(b \wedge a) = (a \wedge b)$	OPEN
L_2	(L_1)	$\vdash$	$(p\ (b \wedge a))$	$=_{\text{subst}}$: $(\langle 1 \rangle)(L_1 L_3)$
C	$()$	$\vdash$	$(p\ (a \wedge b)) \Rightarrow (p\ (b \wedge a))$	$\Rightarrow_I$: (L_2)

Example ND-Proof

Problem: $p_{o\to o}\,(a_o \wedge b_o) \Rightarrow (p\,(b \wedge a))$

ND-proof:

L_1	(L_1)	$\vdash$	$(p\,(a \wedge b))$	Hyp
L_4	(L_1)	$\vdash$	$(b \wedge a) \Leftrightarrow (a \wedge b)$	OPEN
L_3	(L_1)	$\vdash$	$(b \wedge a) = (a \wedge b)$	$\Leftrightarrow$2$=$: (L_4)
L_2	(L_1)	$\vdash$	$(p\,(b \wedge a))$	$=_{\text{subst}}$: $(\langle 1\rangle)(L_1 L_3)$
C	$()$	$\vdash$	$(p\,(a \wedge b)) \Rightarrow (p\,(b \wedge a))$	$\Rightarrow_I$: (L_2)

Example ND-Proof

Problem: $p_{o\to o}\,(a_o \wedge b_o) \Rightarrow (p\,(b \wedge a))$

ND-proof:

L_1	(L_1)	$\vdash$	$(p\,(a \wedge b))$	Hyp
L_4	(L_1)	$\vdash$	$(b \wedge a) \Leftrightarrow (a \wedge b)$	OTTER
L_3	(L_1)	$\vdash$	$(b \wedge a) = (a \wedge b)$	$\Leftrightarrow$2= : (L_4)
L_2	(L_1)	$\vdash$	$(p\,(b \wedge a))$	$=_{\text{subst}}$: $(\langle 1 \rangle)(L_1\,L_3)$
C	$()$	$\vdash$	$(p\,(a \wedge b)) \Rightarrow (p\,(b \wedge a))$	$\Rightarrow_I$: (L_2)

Partial Argument Instantiations

rule, tactic, method | invoking command

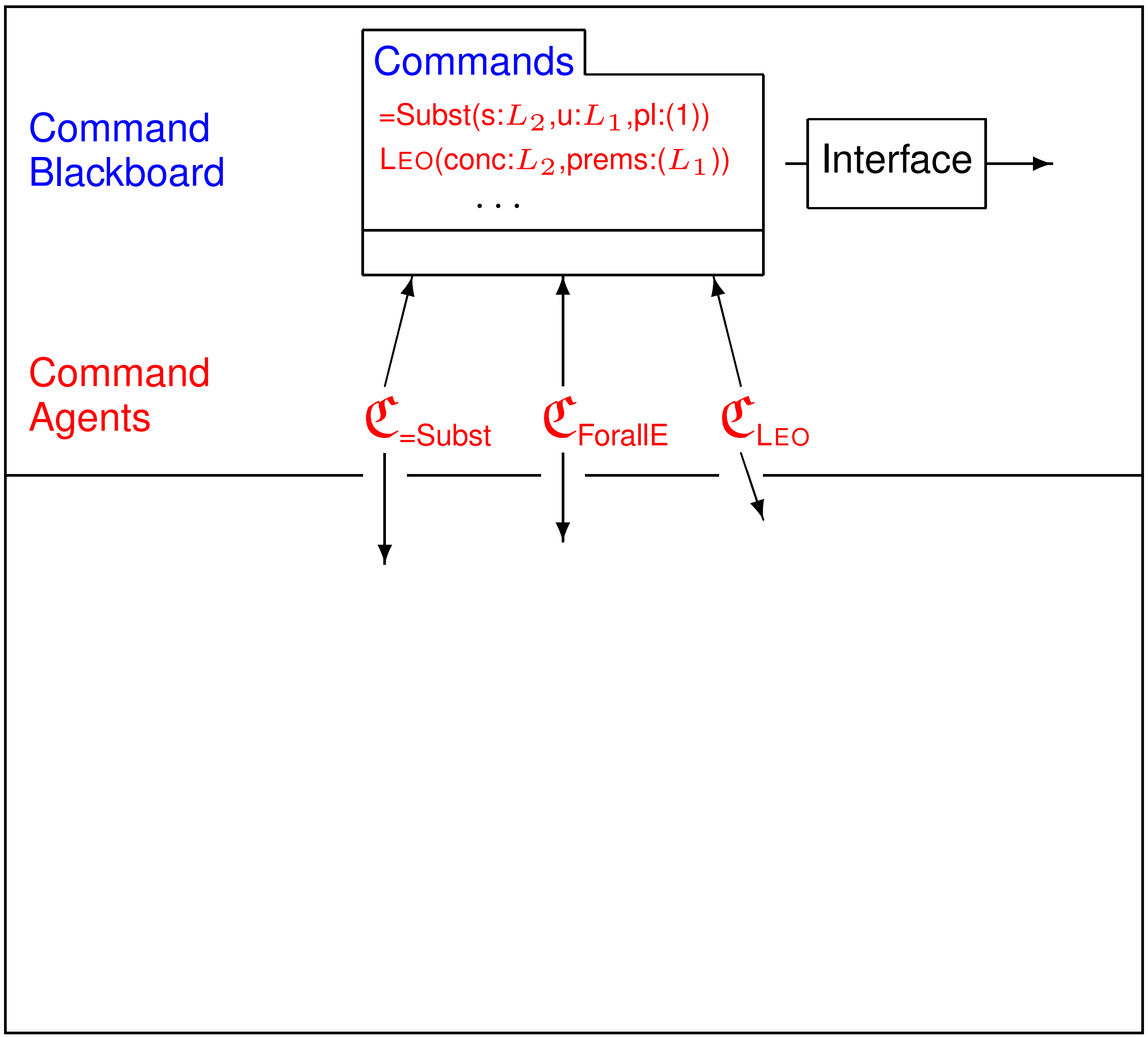
$$\frac{\Phi[x]_p \quad x = y}{\Phi'[y]_p} =_{Subst}(p) \quad \longrightarrow \quad \frac{prem \quad eq}{conc} \text{=Subst}(pos)$$

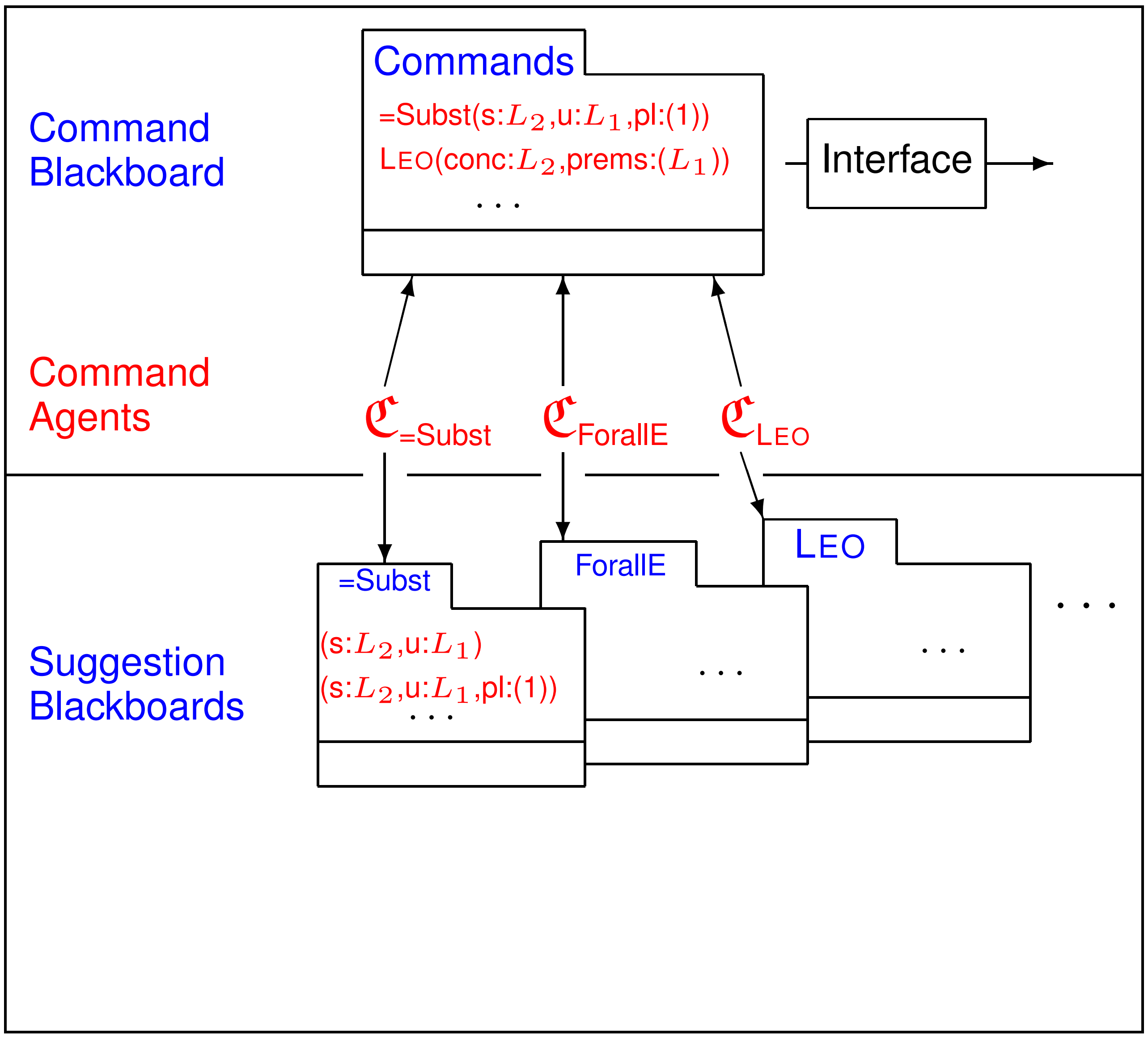
$$\frac{\forall x. A}{[t/x]A} \forall_E(t) \quad \longrightarrow \quad \frac{prem}{conc} \text{ForallE}(term)$$

- Representing suggestions by Partial Argument Instantiations (PAI)

$$\text{=Subst}(prem : L_1,\ conc : L_2,\ eq : L_3,\ pos : (1))$$
$$\text{=Subst}(prem : L_1,\ eq : L_3,\ pos : (1))$$
$$\text{ForallE}(prem : L_4)2$$

- Task of the suggestion mechanism: compute PAI's


Commands
=Subst(s:L_2,u:L_1,pl:(1))
LEO(conc:L_2,prems:(L_1))
. . .
Interface
Command
Blackboard
Command
Agents
$\mathfrak{C}_{\text{=Subst}}$
$\mathfrak{C}_{\text{ForallE}}$
$\mathfrak{C}_{\text{LEO}}$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Interface
Command
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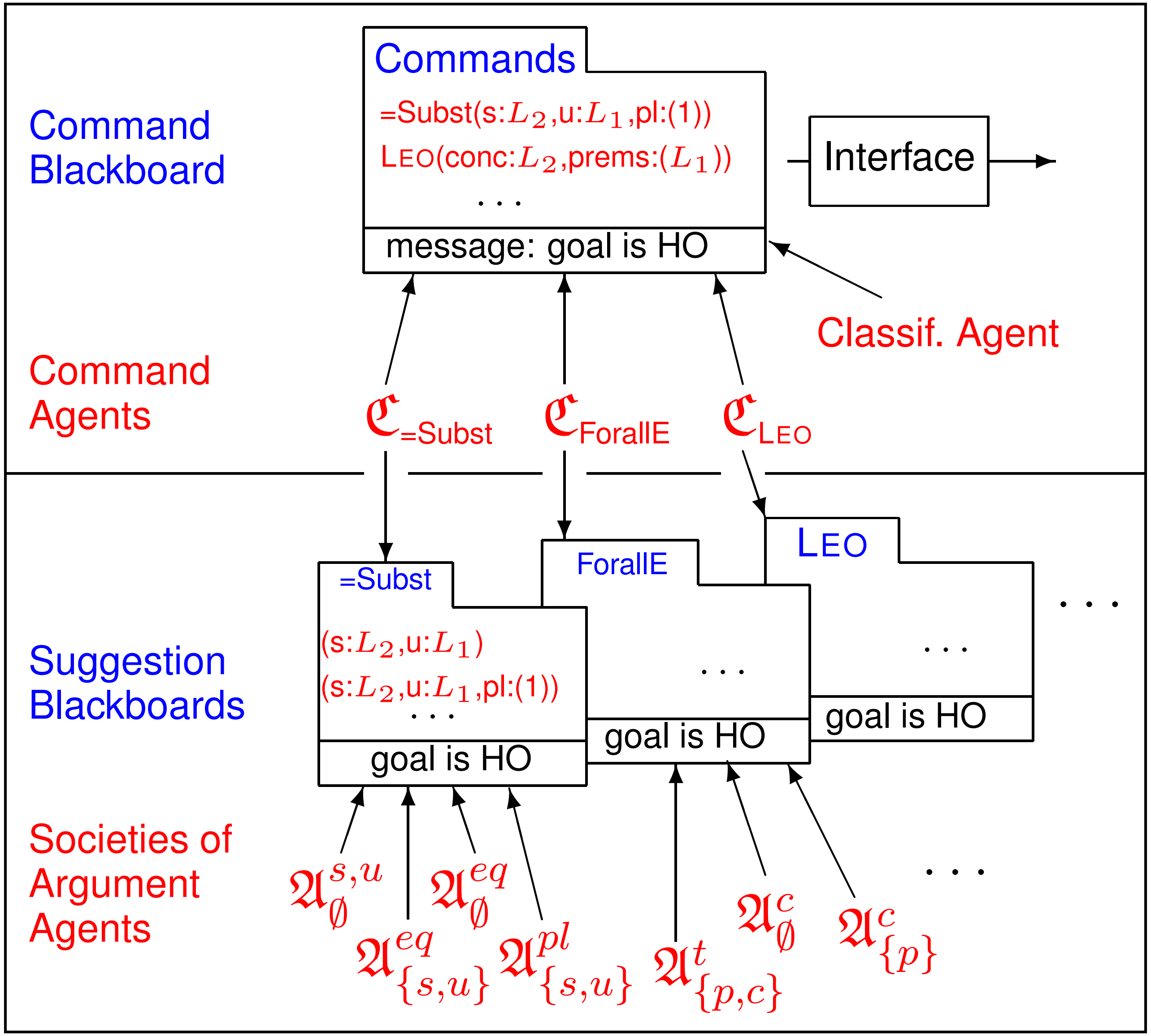
Suggestion
Blackboards
=Subst
(s:L_2,u:L_1)
(s:L_2,u:L_1,pl:(1))
...
ForallE
...
LEO
...
...



Commands
=Subst(s:L_2,u:L_1,pl:(1))
LEO(conc:L_2,prems:(L_1))
. . .
message: goal is HO
Interface
Command Blackboard
Classif. Agent
Command Agents
$\mathfrak{C}_{\text{=Subst}}$
$\mathfrak{C}_{\text{ForallE}}$
$\mathfrak{C}_{\text{LEO}}$
=Subst
(s:L_2,u:L_1)
(s:L_2,u:L_1,pl:(1))
. . .
goal is HO
ForallE
. . .
goal is HO
LEO
. . .
goal is HO
. . .
Suggestion Blackboards
Societies of Argument Agents
$\mathfrak{A}^{s,u}_{\emptyset}$
$\mathfrak{A}^{eq}_{\emptyset}$
$\mathfrak{A}^{eq}_{\{s,u\}}$
$\mathfrak{A}^{pl}_{\{s,u\}}$
$\mathfrak{A}^{t}_{\{p,c\}}$
$\mathfrak{A}^{c}_{\emptyset}$
$\mathfrak{A}^{c}_{\{p\}}$
. . .

Argument Agents

ForallE-Agents: $$\frac{\forall x.\, A}{[t/x]A}\ \forall_E(t) \quad \longrightarrow \quad \frac{prem}{conc}\ \text{ForallE}(term)$$

$$\begin{aligned}
\mathfrak{A}^{prem}_{\emptyset} &= \{\text{find universally quantified line } prem\} \\
\mathfrak{A}^{conc}_{\{prem\}} &= \{\text{find `matching' open line } conc \text{ for } prem\} \\
\mathfrak{A}^{term}_{\{prem,conc\}} &= \{\text{compute mgm(scope of } prem,\ conc)\} \\
\ldots &
\end{aligned}$$

Argument-Agents

$=$Subst-Agents: $\dfrac{\Phi[x]_p \quad x = y}{\Phi'[y]_p} =_{Subst}(p) \quad \longrightarrow \quad \dfrac{prem \quad eq}{conc}$ $=$Subst(pos)

$$\mathfrak{A}_{\emptyset}^{eq} = \{\text{find an equality line } eq\}$$

$$\mathfrak{A}_{\emptyset}^{conc,prem} = \left\{\begin{array}{l}\text{find open line } conc \text{ and a support line } prem \text{ that differ} \\ \text{only wrt. occurences of a single proper subterm}\end{array}\right\}$$

$$\mathfrak{A}_{\{conc,prem\}}^{eq} = \{\text{find equality line } eq \text{ suitable for rewriting } prem \text{ into } conc\}$$

$$\mathfrak{A}_{\{prem,conc\}}^{pos} = \{\text{compute positions where } conc \text{ and } prem \text{ differ}\}$$

... currently 25 agents specified for $=_{Subst}$ ...

Argument Agents at Work

A_1	(A_1)	$\vdash$	$\forall X_o \blacksquare p_{o\to o}(b_o \wedge X_o)$	Assumption
A_2	(A_2)	$\vdash$	$\forall Y_o \blacksquare p_{o\to o}(Y_o \wedge c_o)$	Assumption
L_1	(L_1)	$\vdash$	$p_{o\to o}(a_o \wedge b_o)$	Hyp
		$\ldots$		
L_2	(L_1)	$\vdash$	$p_{o\to o}(b_o \wedge a_o)$	OPEN
C	$()$	$\vdash$	$p_{o\to o}(a_o \wedge b_o)) \Rightarrow p_{o\to o}(b_o \wedge a_o))$	$\Rightarrow_I$: (L_2)

=Subst
()

ForallE
()

A_1	(A_1)	$\vdash$	$\forall X_o \blacksquare\, p_{o\to o}(b_o \wedge X_o)$	Assumption
A_2	(A_2)	$\vdash$	$\forall Y_o \blacksquare\, p_{o\to o}(Y_o \wedge c_o)$	Assumption
L_1	(L_1)	$\vdash$	$p_{o\to o}(a_o \wedge b_o)$	Hyp
		$\dots$		
L_2	(L_1)	$\vdash$	$p_{o\to o}(b_o \wedge a_o)$	OPEN
C	$()$	$\vdash$	$p_{o\to o}(a_o \wedge b_o)) \Rightarrow p_{o\to o}(b_o \wedge a_o))$	$\Rightarrow_I$: (L_2)

$\mathfrak{A}_{\emptyset}^{prem,conc}$

=Subst
$(prem : L_1,\ conc : L_2)$ $()$

$\mathfrak{A}_{\emptyset}^{prem}$

ForallE
$(prem : A_1)$ $(prem : A_2)$ $()$

A_1	(A_1)	$\vdash$	$\forall X_o \centerdot p_{o\rightarrow o}(b_o \wedge X_o)$	Assumption
A_2	(A_2)	$\vdash$	$\forall Y_o \centerdot p_{o\rightarrow o}(Y_o \wedge c_o)$	Assumption
L_1	(L_1)	$\vdash$	$p_{o\rightarrow o}(a_o \wedge b_o)$	Hyp
		$\dots$		
L_2	(L_1)	$\vdash$	$p_{o\rightarrow o}(b_o \wedge a_o)$	OPEN
C	$()$	$\vdash$	$p_{o\rightarrow o}(a_o \wedge b_o)) \Rightarrow p_{o\rightarrow o}(b_o \wedge a_o))$	$\Rightarrow_I: (L_2)$

$\mathfrak{A}^{pos}_{\{prem,conc\}}$

=Subst
$(prem : L_1,\ conc : L_2,\ pos : (1))$
$(prem : L_1,\ conc : L_2)$
$()$

$\mathfrak{A}^{conc}_{\{prem\}}$

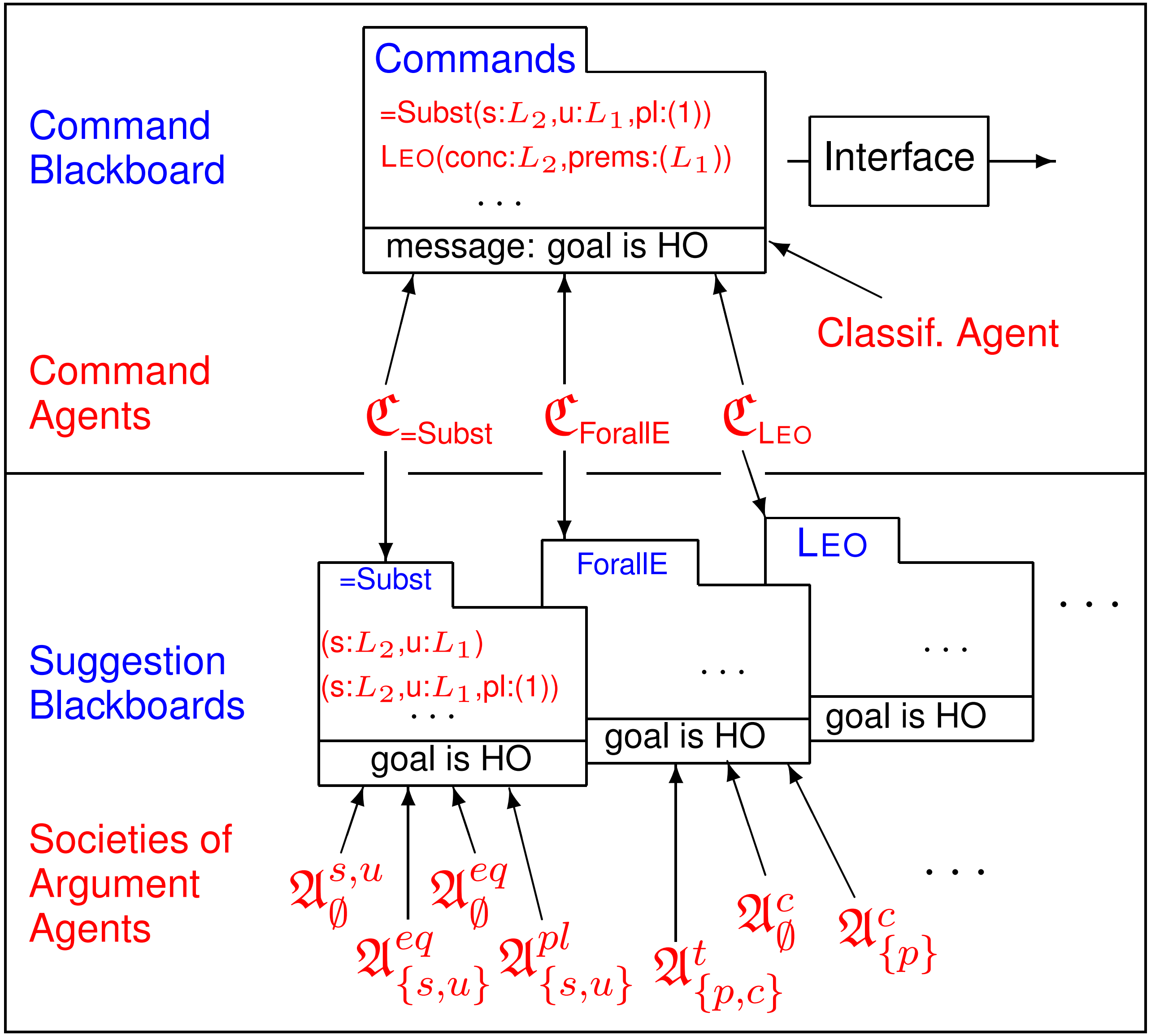
ForallE
$(prem : A_1,\ conc : L_2)$
$(prem : A_1)$
$(prem : A_2)$
$()$

A_1	(A_1)	$\vdash$	$\forall X_o\centerdot p_{o\to o}(b_o \wedge X_o)$	Assumption
A_2	(A_2)	$\vdash$	$\forall Y_o\centerdot p_{o\to o}(Y_o \wedge c_o)$	Assumption
L_1	(L_1)	$\vdash$	$p_{o\to o}(a_o \wedge b_o)$	Hyp
		$\ldots$		
L_2	(L_1)	$\vdash$	$p_{o\to o}(b_o \wedge a_o)$	OPEN
C	$()$	$\vdash$	$p_{o\to o}(a_o \wedge b_o)) \Rightarrow p_{o\to o}(b_o \wedge a_o))$	$\Rightarrow_I : (L_2)$

=Subst
$(prem : L_1,\ conc : L_2,\ pos : (1))$
$(prem : L_1,\ conc : L_2)$
$()$

$\mathfrak{A}^{term}_{\{prem,conc\}}$

ForallE
$(prem : A_1,\ conc : L_2,\ term : a_o)$
$(prem : A_1,\ conc : L_2)$
$(prem : A_1)$
$(prem : A_2)$
$()$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$\mathfrak{A}^{c}_{\{p\}}$
...

A Resource Concept is needed

- In large proof contexts hundreds of agents may become active
- They may employ quite expensive algorithms
  - higher-order unification/matching
  - comparing all subterm positions of terms
  - employ little deductive processes or external reasoning systems
  - ...
- The suggestion mechanism should support different modi
  - quick respond mode $\longleftrightarrow$ lunch time mode

A Resource Adapted Approach

- Complexity rating γ (mirrors computation costs): e.g.

$$\gamma(\mathfrak{A}_{\emptyset}^{conc,prem}) = 10$$

and for command $\frac{prem \quad eq}{conc}$ =Subst(pos)

$$\gamma(\mathfrak{A}_{\emptyset}^{eq}) = 2$$

- User specifies a global deactivation threshold in order to suppress computations of 'expensive' agents

Disadvantages:

Tuning by hand not reflects real costs, no dynamic adjustments, tuning of hundreds of single argument agents

A Resource Adaptive Approach

1. Self adjustments of the argument agents

- Argument agents monitor and adjust their own performance and contributions in the past using as criterions:
  - cpu time the argument agents consume
  - penalty for failing several times in a row
- Resource adjustments are communicated to command agents via blackboards

A Resource Adaptive Approach

2. Explicit reasoning on suitable resource distributions

- The command agents accumulate the ratings of their argument agents and pass them to a special resource agent via the command blackboard
- Resource agent reasons on global adjustments and communicates them to the lower layer agents via the blackboards
- Aspects of the resource reasoning
  - avoid complete retirement
  - user specific preference for certain commands
  - switch of proof context
  - . . . and . . .

A Resource Adaptive Approach

3. Informed activation & deactivation

- Rules, tactics, methods are associated with a specific logic or theory

  (ΩMEGA provides special proof methods for limit-theorems)

- Problems (in mathematics) are associated with a specific logic or theory

  (Extensionality rules belong to HO, ForallE belongs to FO, AndE belongs to PL)

$\Rightarrow$ Use knowledge to activate appropriate and deactivate inappropriate agents

- A classification agent tries to classify each new subproblem and passes this information via the blackboards to the argument agents

- Argument agents can themselves decide whether they belong to this class and get active or inactive

Applications

- Suggestion mechanism for interactive systems
  - suggest next interactive step
  - complete partial command instantiations suggested by the user
- Support in proof planning: compute applicable proof methods in parallel
- Suggestion mechanism enriched with simple backtracking over the suggestions = an automated theorem prover (proof planner)

Conclusion

Advantages of the proposed suggestion mechanism to other systems:

- Flexibility
- Anytime character
- Robustness
- Expandability, User adaptability, Manageability

What interactive theorem provers gain:

- Better exploitation of available resources as in traditional systems (steadily works background, autonomous, concurrent, resource adaptive)