Semantics and Automation of Higher-Order Logic

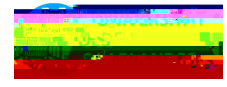
– Some Remarks –

Christoph Benzmüller

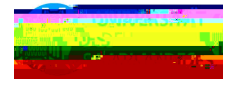
[illegible]

Workshop on Logic, Proofs and Programs

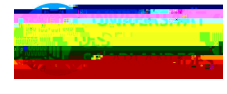
17–18 June 2004, Nancy



First-order Logic



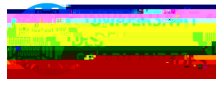
Higher-Order Logic

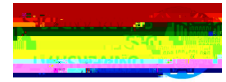


Motivation for Talk

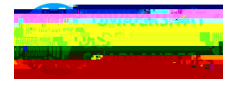
Is the situation really hopeless?

Is it justi able that the deduction

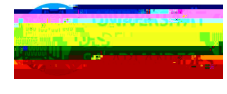




HOL: Classical Type Theory



HOL: Semantics

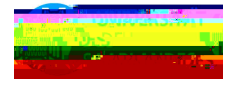


Sidetrack: Logical Frameworks

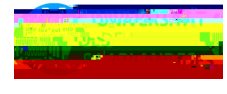
Presentation by

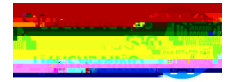
Marc Wagner

Logical Frameworks



Exercise Sheet III





HOL Semantics: Applications

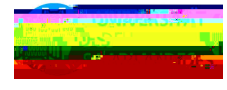
Henkin semantics

Mathematics

Without Boolean extensionality

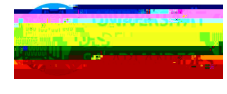
Linguistics, intensional contexts

“I believe



HOL: Problems

Problem 2elln2suita(lem)Tj /R8941 0 Td (lems)Tj29063 0 Td proofPr

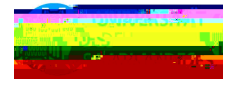


Abstract Consistency

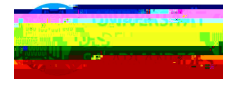
Completeness proofs in HOL much harder than in FOL

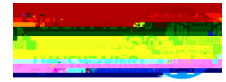
Direct semantical arguments are too complicate

Abstract consistency proof



Abstract Consistency





ND Calculi: Completeness

Excerpt from completeness proof . . .

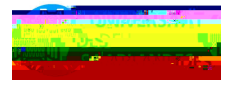
r$_\beta$: Let A 2 and A$\dot{\forall}^?_\beta$ be NK -inconsistent. That is, A$\dot{\forall}^?_\beta$ u l B$_o$ / R 1 7 1

By NK : l , we know ` : A$\dot{\forall}^?_\beta$. Since A$_\beta$ by

NK Hyp /R179 ET Q 1 0 0 RG 1 0 0 rg q 10 0 0 10 0 0 cm BT /R298 1 Tf 0 2(

^{o}B^2

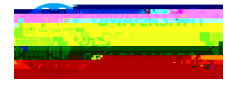
00457 0 Td (b)Tj 0.539576 0 Td (ut)Tj 1.10215 0 Td (both)Tj /



Saturation and Cut

Saturation condition r_{sat} is a challenge for machine-oriented calculi:

- **as hard as cut-elimination**
- **therefore development of alternative, weaker conditions in [BenzmüllerBrownKohlhase-Draft03] which are**



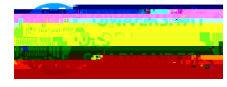
Problem 3:

The two crucial challenges for automation of HOL

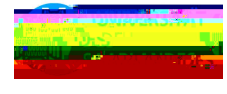
- treatment of equality and extensionality
- instantiation of set variables

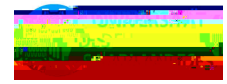
are too hard to control successfully.

Really?



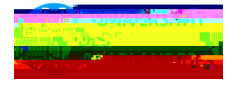
Extensional





Extensional Resolution

Further small examples



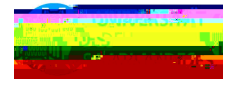
Sidetrack: Lambda Cube

Presentation by

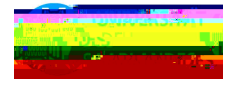
Matthias Berg

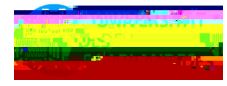
Lambda Cube

See extra



Sidetrack: New Foundations





Difference Reduction

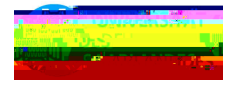
Extensional RUE-resolution

[Benzmüller-PhD-99]

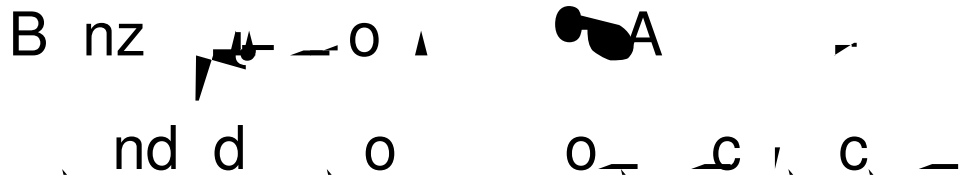
Difference reduction matrix calculus

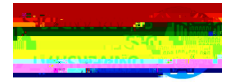
[Brown-PhD-04]

- **Alcrules for extensional resolution**
- **Positive extensionality rules, but no paramodulation rule**
- **New: Resolution and factorization allowed on uni cation constraints**



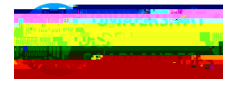
Prover LEO

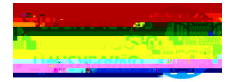




Superposition with Equivalences

$$\frac{l = r \;\_\; C}{l}$$

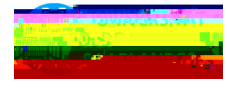




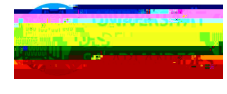
Remark

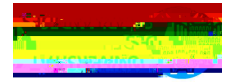
More useful as restricted extensions of FOL approaches: Embedding/Implementation of FOL approaches in HOL context?

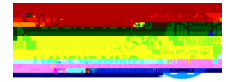
Very important: Extension of CASC competition and TPTP library in order to avoid isolated analysis of FOL approaches.



Example: As an exampleHOL







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