Automated Theorem Proving in First-Order and Higher-Order Logic

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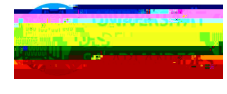
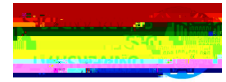
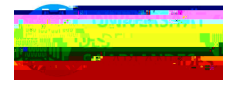
Lecture Course

Saarbrücken, Germany







Module Outline (To be discussed)

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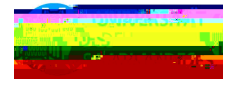
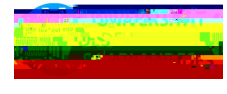
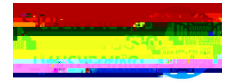


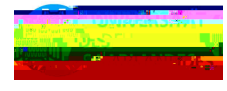
Pre-Test

Take a sheet of paper and try to answer the following questions:

1. **Encode the following statement in a set of propositional logic formulas S:**







History (Cont'd)

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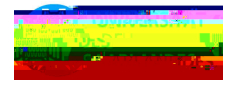

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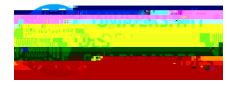
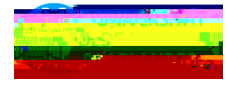


History (Cont'd)

Hilbert's progr



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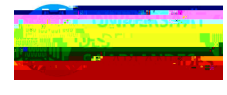

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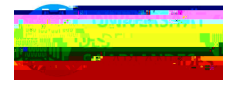
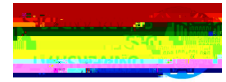
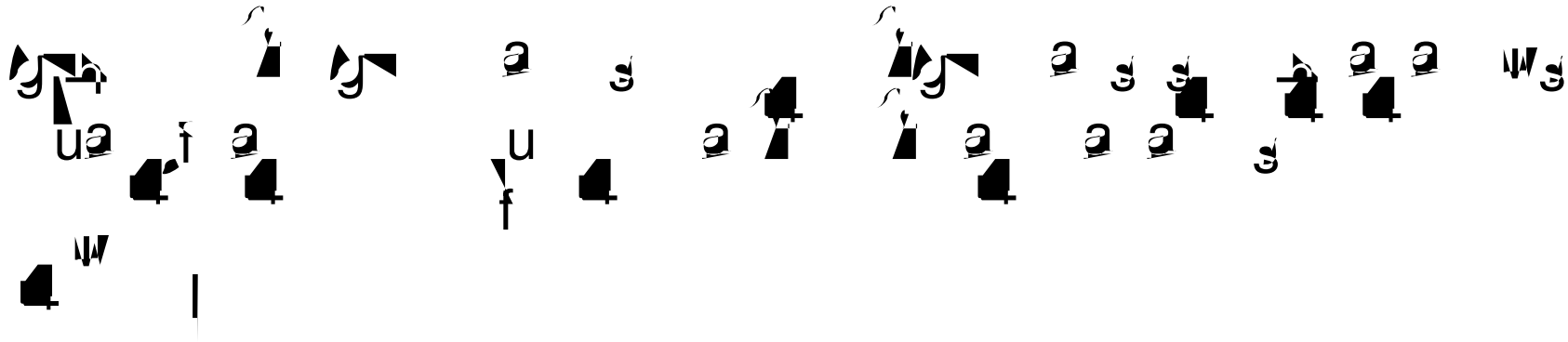
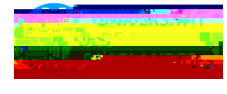


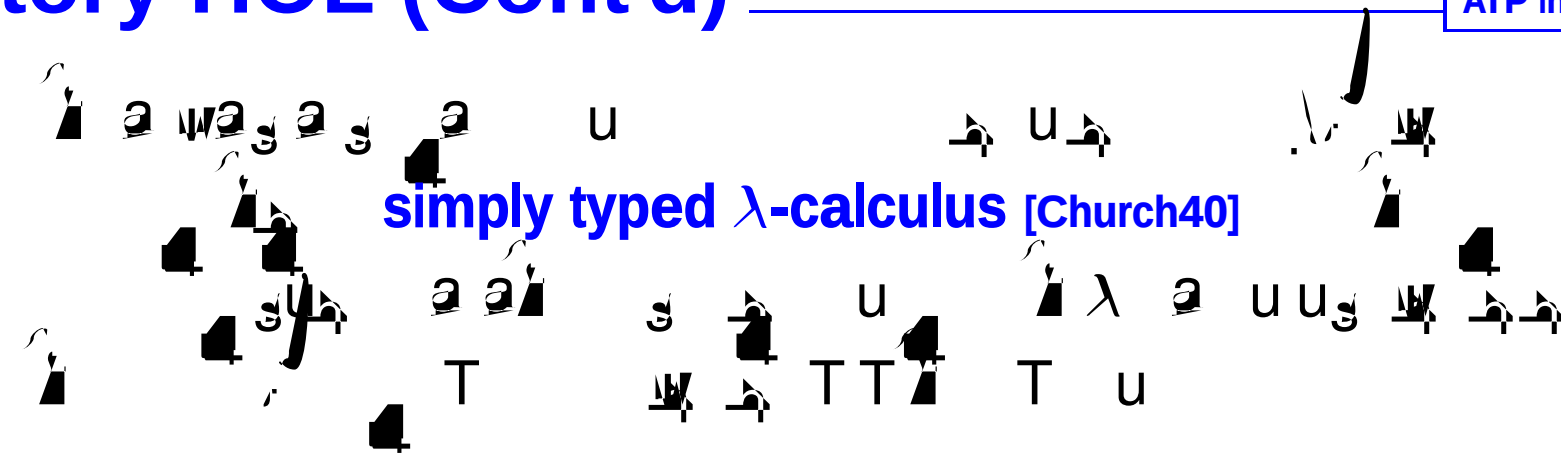


History HOL

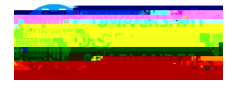




History HOL (Cont'd)



History HOL (Cont'd)



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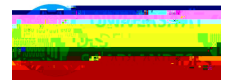
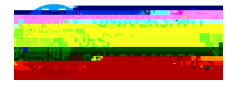


Propositional Logic ($\mathcal{P}$): Syntax

$\mathcal{P}$: Syntax (Cont'd)



Our Pre-Test: Solution

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is mammal if it is horned.

$S = \{mo, my, ma, ho\}$

(1) $my \quad \neg mo$

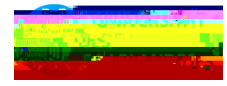
(2) $\neg my \quad (mo \quad ma)$

(3) $(\neg mo \blacktriangledown ma) \quad ho$

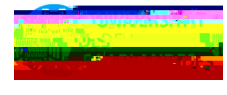
(4) $ho \quad ma$

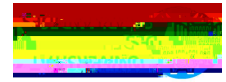
(1)$\neg my \blacktriangledown \neg mo$, (2a)$\neg\neg my \blacktriangledown \neg(\neg mo \blacktriangledown \neg ma)$,

(3)$\neg(\neg mo \blacktriangledown ma) \blacktriangledown ho$, (4)$\neg ho \blacktriangledown ma$

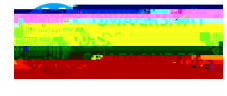


$\mathcal{P}$: Structural Induction



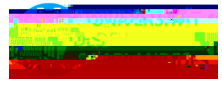


$\mathcal{P}$: Semantics (Cont'd)



$\mathcal{P}$: Semantics (Cont'd)

Remark 1.11 **I_V is a total, terminating, and wd (tot 32 44 0 Td ell-de ned,)Tj 4885659 0 Td fu**



$\mathcal{P}$: Semantics (Cont'd)

De nition 1.13 (Satis ability and Validity) **A formula**

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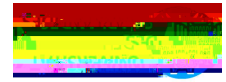
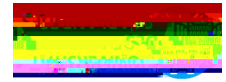
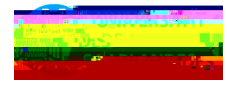
$\mathcal{P}$: Semantics (Cont'd)





$\mathcal{P}$: Resolution (Cont'd)



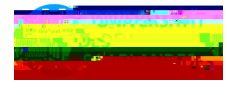
$\mathcal{P}$: Resolution (Cont'd)

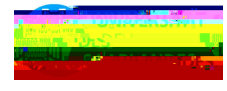
De nition 1.22 **We de ne a mapping form2lit : $\mathcal{P}$, where L describes the set of literals:**

$$\text{form2lit}(A) = \begin{cases} [B]^F & \text{if } A \text{ has form } \neg B \text{ for some formula } B \\ \vdots & \end{cases}$$

[illegible]

$\mathcal{P}$: Resolution (Cont'd)





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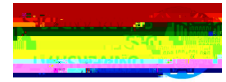
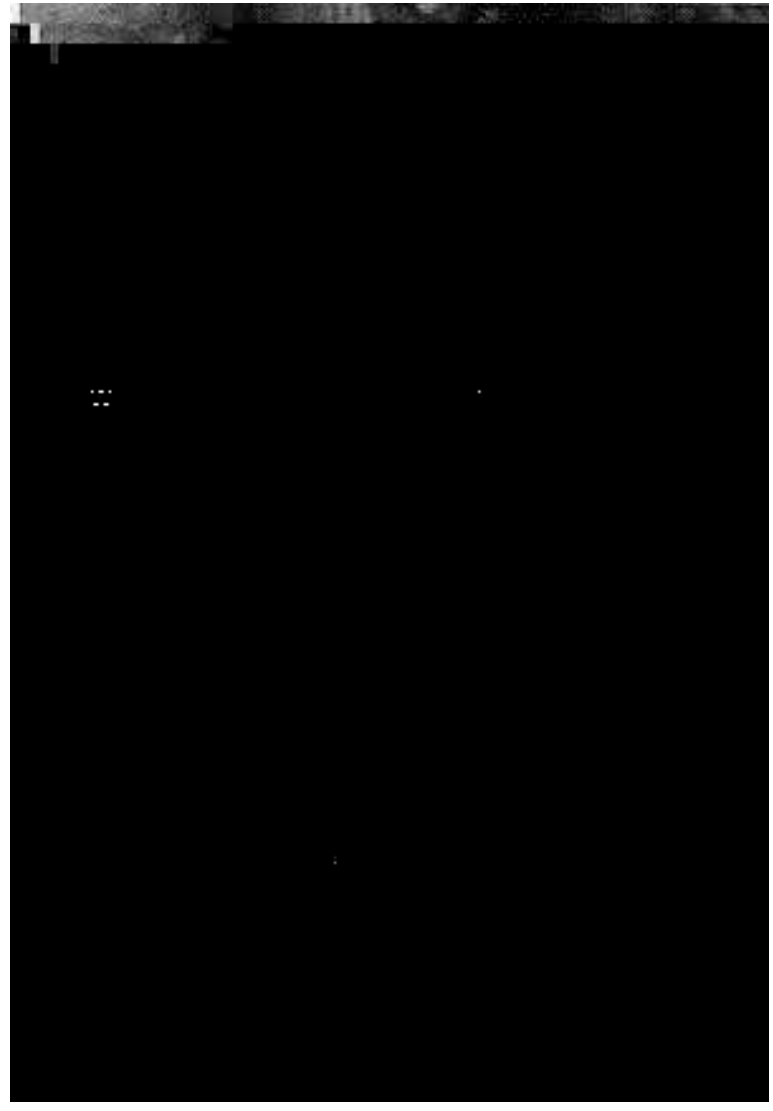
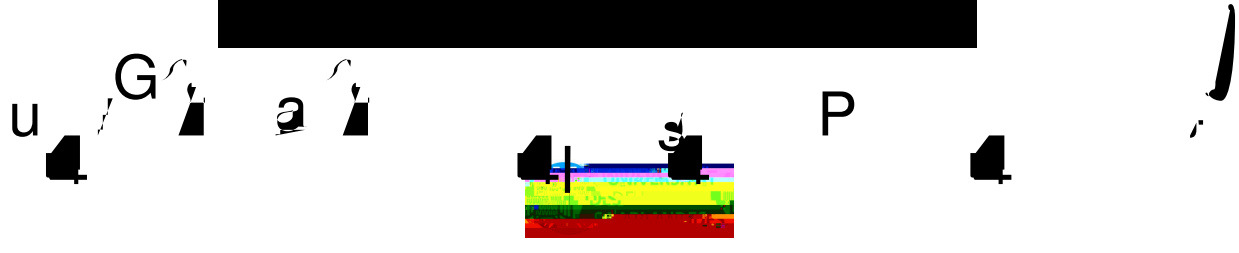


Sidetrack: Kurt Gödel



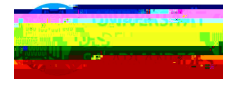


Gödels Theorem's: Links

www.ltn.lv/~podnieks/

kilby.stanford.edu/~rvg/154/handouts/incompleteness.html

en.wikipedia.org/wiki/Talk:



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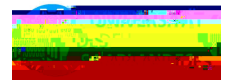

$\mathcal{P}$: Resolution (Cont'd)

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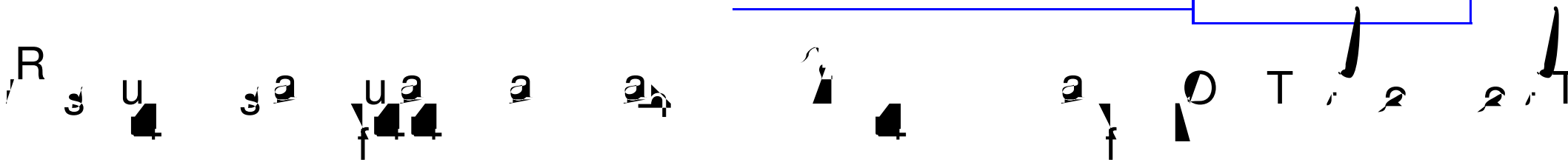
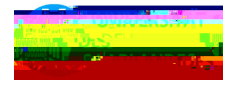


$\mathcal{P}$:



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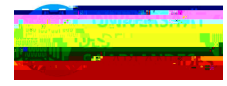

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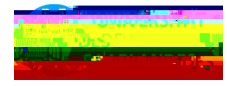

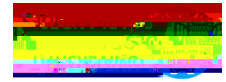

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Sidetrack: Saturate

Presentation by Ruzica Piskac

The Saturate System

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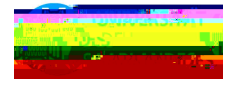
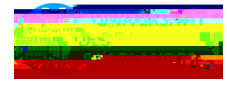
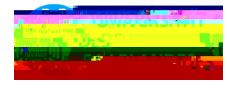
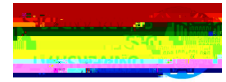


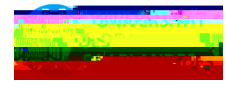
Abstract Consistency: History



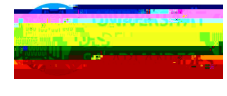
Abstract Consistency



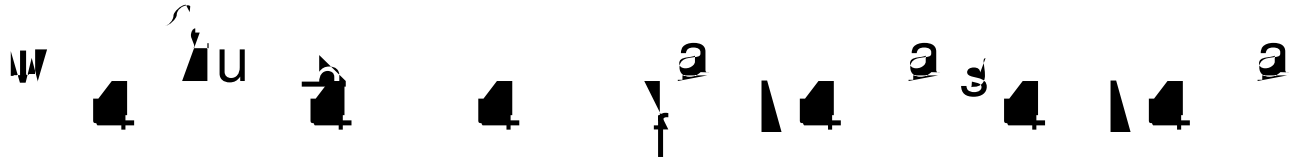




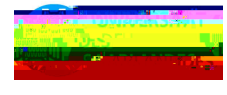
Abstract Consistency (Cont'd)



Abstract Consistency (Cont'd)



Abstract Consistenc

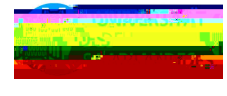


Abstract Consistency (Cont'd)

Lemma 1.66 (Hintikka Extension Lemma) **Let be a compact abstract consistency class and let an element of this class. Then there exists a Hintikka set H for , such that H.**

Proof: **For a given ϕ we construct H according to the following de nition:**

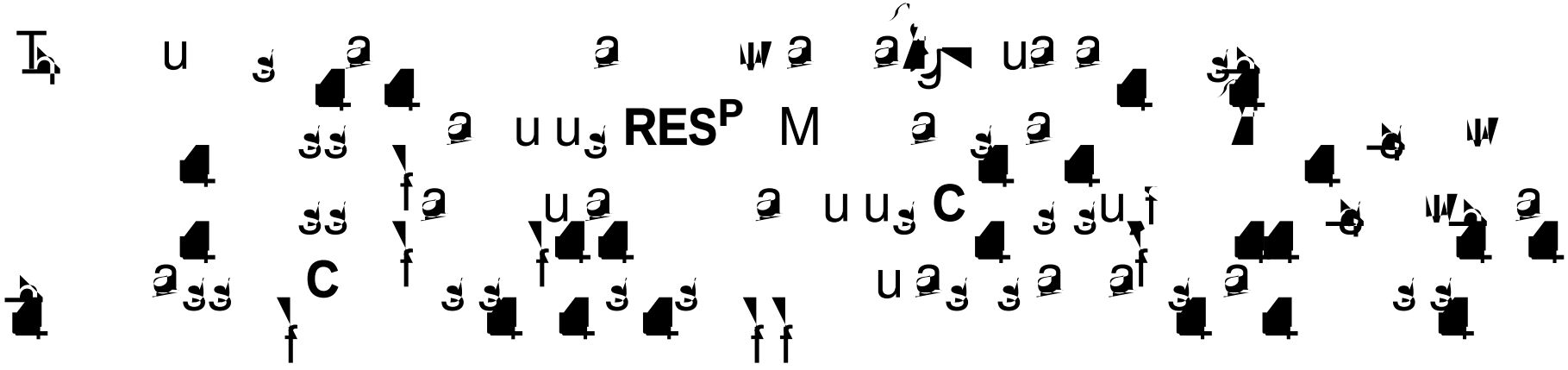
H



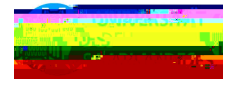
Abstract Consistency (Cont'd)

Theorem 1.67 (Model Existence Theorem) **Let be a saturated propositional abstract consistency class and**

Abstract Consistency (Cont'd)



Abstract Consistency (Cont'd)



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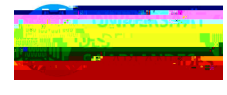

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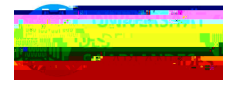

Sidetrack: Isabelle/HOL

Presentation by Andrey Shadrin

Isabelle-HOL

See extra slides