

Computerized Verification of Formal Reconstructions of Anselm's Argument by Eder and Ramharter

Lukas Grätz, Fabian Schütz

Project of Computational Metaphysics 2016

C. Benz Müller, A. Steen, M. Wisniewski

Freie Universität Berlin, Institute of Computer Science, Germany



Introduction

Formal reconstruction of arguments given in natural language is a highly non-trivial task. It requires deep discussion about what is being said and considerable knowledge of appropriate formal languages. But success is greatly rewarded. Theorem provers such as PVS or Isabelle can verify consistency of highly complex proposition systems or find a minimal set of premises.

Timeline

- 1078 Anselm of Canterbury first proposes an ontological argument in his work Proslogion
- 2015 Eder and Ramharter give several modal and non-modal formalizations in Formal reconstructions of St. Anselm's ontological argument
- 2015 John Rushby verifies some of Eder and Ramharter's formalizations using the PVS proofer system
- 2016 Grätz, Kloht, Deisel and Schütz verify all formalisations in Isabelle

Requirements of formal reconstructions

According to Eder and Ramharter [1] a formal reconstruction should ...

- (1) conform with what the author said,
- (2) be maximally compatible with what the author said elsewhere,
- (3) represent the fundamental structure of the argument,
- (4) clearly separate between initial conformity and second-step improvements,
- (5) include premisses that contain the conclusion in a *non-obvious way*, and
- (6) attribute to the author only premisses that he could have held to be true for conceptual (non-empirical) reasons.
- (7) Also, the premisses should be plausible from the standpoint of the author.

Modern rendition of Anselm's argument

We can conceive of something than which there is no greater.

If that thing exists only in the mind and not in reality, then we can conceive of something greater. Namely, a similar thing that does exist in reality.

Therefore either the greatest thing exists in reality or it is not the greatest thing.

Therefore the greatest thing necessarily exists in reality.

Thats God!

Non-Modal Formalization

God: $God \equiv \lambda x. \neg(\exists y. (y \mathbf{G} x))$

ExUnd: $\exists x. God\ x$

G (informally): To evaluate $y \mathbf{G} x$, x and y are compared relative to properties (predicates) that are elements of a given list \mathbf{P} containing all *positive* properties. $y \mathbf{G} x$ holds, if and only if y satisfies all positive properties that x satisfies and at least one that x does not satisfy.

Realization (informally): Each subset of \mathbf{P} can be realised. That is, for each subset p of \mathbf{P} there exists some

entity x that satisfies exactly the properties in p .

P(re): Existence in reality (**re**), as apposed to merely conceivable existence is a positive property and thus **P(re)**.

Notable about this formalization is the idea of some set P of *positive properties*. The textual argument suggests that some y be greater than x if y and x are equal except for only y satisfying a property **re** of real existence. Generalizing on this idea **re** can be thought of as one of many *greater-making properties*. However the assumption *Realization* appears uncalled for.

Verification

Proof reconstruction: The Isabelle/HOL prover allows for a proof by case distinction that is similar to the one given in [7]. The full proof can be seen in [5].

Proof without ExUnd: Using automated theorem provers, a second proof was found that does not need the assumption of **ExUnd**. The key to understand this proof is the insight that realization of **P** yields an x that has all the intended properties.

Conclusion I

Realization is too strong and uncalled for as an assumption. It is doubtful whether point (1) of Eder and Ramharter's requirements is met in this case. Moreover, *Realization* deforms the proof in way such that it cannot be called *ontologic* as *real existence* and *conceivable existence* fall together in a single assumption. This seems to contradict point (5).

Modal Formalization

Eder and Ramharter's modal reconstruction can be formalized in Isabelle/HOL (cf. Fig. 1). Note the additional assumption **PossEx**.

```
7 consts Mre :: "μ ⇒ σ"
8
9 abbreviation PossEx where
10 "PossEx ≡ [∀x. ∃(Mre x)]"
11
12 consts greatness :: "μ ⇒ 1 ⇒ μ"
13
14 abbreviation meq :: "(1 ⇒ μ) ⇒ (1 ⇒ μ) ⇒ σ" (infix "≡" 52)
15 where "x ≡ y ≡ λw. x w = y w"
16
17 consts gr :: "μ ⇒ μ ⇒ σ" (infix "≡" 54)
18
19 abbreviation Mgr :: "(1 ⇒ μ) ⇒ (1 ⇒ μ) ⇒ σ" (infix "≡" 54)
20 where "p ≡ q ≡ λw. (p w ≡ q w)"
21
22 abbreviation MGod where
23 "MGod ≡ λx. ∃z. (λw. z) =≡ greatness x ∧ ¬∃y. greatness y ≡≡ (λw. z)"
24
25 consts "P" :: "(μ ⇒ σ) ⇒ bool"
26
27 abbreviation P_Mre where
28 "P_Mre ≡ P Mre"
29
30 abbreviation Greater where
31 "Greater ≡ [∀ x y. (∃F. (λw. P F) ∧ ¬(F x) ∧ ∅(F y))
32 ↔ (∃z. (λw. z) =≡ greatness x ∧ ∅(greatness y ≡≡ (λw. z)))]"
33
34 theorem GodE:
35 assumes "[∃ x. ¬ MGod x]"
36 assumes PossEx
37 assumes Greater
38 assumes P_Mre
39 shows "[∃x. MGod x ∧ Mre x]" using assms(1) assms(2) assms(3) assms(4) by fastforce
40
```

Figure 1: Isabelle/HOL code of modal reconstruction

Revision by Charles Hartshorne

For his proof, Hartshorne assumes an **S5** modal logic - that is one, where the accessibility relation is both reflexive and euclidean. As we will later see, this seems reasonable.

Anselm's Principle: $p \rightarrow \Box p$
where p is a predicate that indicates that "a perfect being exists"

Intuitive Postulate: $\Diamond p$
where p is again the predicate that indicates that "a perfect being exists"

Proof rendition by Joshua Ernst [2]

1. $\Box p \vee \neg \Box p$ excluded middle
2. $\neg \Box p \rightarrow \Box \neg \Box p$ Becker's postulate
3. $\Box p \vee \Box \neg \Box p$ 1, 2
4. $\Box \neg \Box p \rightarrow \Box \neg p$ Anselm's principle
5. $\Box p \vee \Box \neg p$ 3, 4
6. $\Box p$ Intuitive postulate, 5
7. p Modal axiom

We formalised this rendition and verified it using p as uninterpreted predicate and also by inserting a specification of p as indicated by Eder and Ramharter [1, 4].

Critique on the modal reconstruction

I) ExUnd restricts model structures. Clearly a world can be conceived that does not have a greatest being, e.g. the natural numbers.

II) Possibility and conceivability are complex concepts revolving around **chance, belief, knowledge and enumeration**. More importantly they are generally distinct. Their precise meaning needs to be properly expressed. For a discussion, see [3].

III) E! in an actual world appears a lot like saying it twice. ExUnd already speaks of existence. The question really is *if ExUnd applies to the actual world or just contrived ones*.

IV) The predicate E! of real existence actually applies to the idea of god. One may rightfully ask what it means that an idea really exists.

V) As Kant objects in his *Critique of Pure Reason* [6] predicated existence is problematic in terms of logic.

VI) We perceive the world as evolving but modal logic does not know a concept of time or change.

VII) It is doubtful given the simple logic that is actually present in Anselm's text that he had in mind something similar to Kripke semantics or a relation of possible worlds.

VIII) None of Eder and Ramharter's reconstructions hold any reference to the fool in Anselm's argument.

Conclusion II

Formal reconstruction of arguments is a standard and often performed task in philosophy. There is a question though what this actually achieves. Should a formal reconstruction of an argument capture its *true meaning*? If so it would make sense to ask how it influences people's attitudes. In this sense all the reconstructions of Anselm's argument that have been created over the centuries are part of its *meaning*. The *true meaning* of Anselm's argument in this sense has not yet fully materialised. This poster you are reading is in the ongoing discussion about it.

References

- [1] Günther Eder and Esther Ramharter. Formal reconstructions of st. anselm's ontological argument. *Synthese*, 192(9):2795–2825, October 2015.
- [2] Joshua Ernst. Charles hartshorne and the ontological argument. *Aporia*, 18(1), 2008.
- [3] George L. Goodwin. *The Ontological Argument of Charles Hartshorne*. PhD thesis, Missoula, Mont., 1978.
- [4] Lukas Grätz and Fabian Schütz. Isabel-hol verification of modal logic formalisations of anselm's ontological argument by eder and ramharter. Technical report, FU Berlin, 2016.
- [5] Lukas Grätz, Fabian Schütz, Tobias Kloht, and Ronja Deisel. Isabel-hol verification of predicate and hol logic formalisations of anselm's ontological argument by eder and ramharter. Technical report, FU Berlin, 2016.
- [6] Immanuel Kant. *Critique of Pure Reason*. Johann Friedrich Hartknoch, 1781.
- [7] John Rushby. Mechanized analysis of a formalization of anselm's ontological argument by eder and ramharter. Csl technical note, Computer Science Laboratory, SRI International, Menlo Park, CA, January 2016.



Leibniz: *Calculus!*

Computational Metaphysics is an interdisciplinary lecture course designed for advanced students of computer science, mathematics and philosophy. The main objective of the course is to teach the students how modern proof assistants based on expressive higher-order logic support the formal analysis of rational arguments in philosophy (and beyond). In our first course in Summer 2016 the focus has been on ontological arguments for the existence of God. However, some students picked formalisation projects also from other areas (including maths).

Computational Metaphysics was awarded the *Central Teaching Award 2015* of the FU Berlin.

