Formalization and Assessment of Lowe's Modal Ontological Argument

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Introduction

We present a formalization of a modal variant of the Ontological Argument put forward by E. J. Lowe [2]. The formalization of this argument has been carried out in an interactive theorem prover (Isabelle/HOL) using two different approaches. Firstly, a modal variant using a modified version of the Quantified Multimodal Logic (QML) embedding [1], and secondly, a simplified variant in classical predicate logic. Our central motivation has been to demonstrate the use of Automated Theorem Proving for clarifying concepts and verifying the validity of complex arguments in philosophy, especially metaphysics, and thus overcoming the limitations of intuitive natural language argumentation.

The Argument

The author (E. J. Lowe) introduces in his article a new modal variant of the Ontological Argument using a classical approach to philosophical argumentation; stating definitions and premises in natural language and then intuitively deducing further statements until finally obtaining the desired conclusion:

- P1 God is, by definition, a necessary concrete being.
- P2 Some necessary abstract beings exist.
- P3 All abstract beings are dependent beings.
- P4 All dependent beings depend for their existence on independent beings.
- P5 No contingent being can explain the existence of a necessary being.
- P6 The existence of any dependent being needs to be explained.
- P7 Dependent beings of any kind cannot explain their own existence.
- P8 The existence of dependent beings can only be explained by beings on which they depend for their existence.

C10 A necessary concrete being exists.

Modal Variant and Actualism

Two main historical developments motivate our modelling choice: (1) There is a long philosophical discussion, which dates back to ancient Greece, about the possibility of speaking about non-existent objects. (2) There is a belief that, in order to avoid the conclusion of the Ontological Argument, one must deny that existence is a property that can be predicated of objects. In this approach we want to circumvent both debates, by introducing actualist quantification to our modal formalization of the Ontological Argument. In a possible world semantics, this amounts to allowing universal and existential quantifiers to range over different domains at distinct possible worlds.

```
abbreviation mforall   :: ('a⇒io)⇒io (∀)
  where ∀Φ ≡ λw.∀x. (ξ x w)⟶(Φ x w)
abbreviation mforallB  :: ('a⇒io)⇒io (binder∀ [8]9)
  where ∀x. φ(x) ≡ ∀φ
abbreviation mexists   :: ('a⇒io)⇒io (∃)
  where ∃Φ ≡ λw.∃x. (ξ x w)∧(Φ x w)
abbreviation mexistsB  :: ('a⇒io)⇒io (binder∃ [8]9)
  where ∃x. φ(x) ≡ ∃φ
```

Figure 1: Embedding of quantifiers in HOL.

Note above that the predicate ξ is part of the meta-language and only used to restrict the domain of quantification. We have no existence predicate in the object language. Nevertheless, we are still able to speak of non-existent objects and prove the Ontological Argument.

```
abbreviation mbox    :: io⇒io (-[52]53)
  where □φ ≡ λw.∀v. (w r v)⟶(φ v)
abbreviation mdia    :: io⇒io (-[52]53)
  where ◇φ ≡ λw.∃v. (w r v)∧(φ v)
abbreviation mactual :: io⇒io (𝓐- [64] 65)
  where 𝓐φ ≡ λw. (φ aw)
```

Figure 2: Embedding of modal operators in HOL.

Since only three out of eight original premises were needed to prove the conclusion, we leave out here many irrelevant concepts and definitions. Moreover, an implicit premise, needed specifically for this modal variant, has also been discovered; *concreteness* must be an essential property of beings.

```
consts Concrete::(e⇒io)
abbreviation Abstract::(e⇒io) where Abstract x ≡ ¬(Concrete x)

consts dependence::(e⇒e⇒io) (infix dependsOn 100)
abbreviation Dependent::(e⇒io) where Dependent x ≡ ∃y. x dependsOn y
abbreviation Independent::(e⇒io) where Independent x ≡ ¬(Dependent x)

axiomatization where
  P2: ⌊∃x. □Abstract x⌋ and
  P3: ⌊∀x. Abstract x → Dependent x⌋ and
  P4: ⌊∀x. Dependent x → (∃y. Independent y ∧ x dependsOn y)⌋ and
  concreteness-is-essential: ⌊∀x. Concrete x → □Concrete x⌋
```

Figure 3: Formalization of Lowe's Ontological Argument in QML.

The main conclusion, stated in natural language, allows for ambiguous interpretations. We formalize and prove some of them:

```
lemma C5: ⌊∃x. Concrete x⌋
using P2 P3 P4 by blast

lemma C10a: ⌊𝓐(∃x. □Concrete x)⌋
using C5 concreteness-is-essential by blast

lemma C10b: ⌊∃x.□Concrete x⌋
using C5 concreteness-is-essential by blast
```

Figure 4: $C5$ is a partial conclusion: In every world there are concrete objects. $C10a$ reads: There are necessarily concrete objects (in the actual world). $C10b$ reads: There are necessarily concrete objects (in every world).

FOL Variant

In the formalization above, we found ourselves eagerly interpreting the argument in a modal context for the sake of honoring the original intention of the author. However, a more literal reading promptly suggests another logical form. For instance, according to the author "there is no logical restriction on combinations of the properties involved in the *concrete/abstract* and the *necessary/contingent* distinctions. In principle, then, we can have contingent concrete beings, contingent abstract beings, necessary concrete beings, and necessary abstract beings."

By taking these four categories as exhausting our domain of discourse, a different reading of *necessity* and *contingency* reveals itself, not as modals, but as mutually exclusive predicates.

As a consequence, our universe of discourse (and some exemplary members) would look as follows:

	Abstract	Concrete
Necessary	*Numbers*	*God*
Contingent	*Fiction*	*Stuff*

```
consts Necessary::(e⇒bool)
abbreviation Contingent :: e ⇒ bool where Contingent x ≡ ¬(Necessary x)
consts Concrete::(e⇒bool)
abbreviation Abstract::(e⇒bool) where Abstract x ≡ ¬(Concrete x)
```

Figure 5: Necessity and contingency modeled as predicates

```
consts dependence::(e⇒e⇒bool) (infix dependsOn 100)
definition Dependent::(e⇒bool) where Dependent x ≡ (∃y. x dependsOn y)
abbreviation Independent::(e⇒bool) where Independent x ≡ ¬(Dependent x)
```

Figure 6: Definitions concerning *Ontological Dependence*

Consequently, the argument can be completely formalized in predicate logic and proved directly in Isabelle/HOL:

```
definition P1-God::(e⇒bool) (God) where God x ≡ Necessary x ∧ Concrete x

axiomatization where
 P2: ∃x. (Necessary x) ∧ (Abstract x)  and
 P3: ∀x. (Abstract x ⟶ Dependent x)  and
 P4: ∀x. (Dependent x ⟶ (∃y. Independent y ∧ x dependsOn y) ) and
 P5: ∀x. (Necessary x ⟶ (∀y. x dependsOn y ⟶ Necessary y) )

lemma C10: ∃x. Necessary x ∧ Concrete x  using P2 P3 P4 P5 by blast
lemma GodExists: ∃x. God x  using C10 P1-God-def by simp
```

Figure 7: Non-modal Ontological Argument formalized in FOL

Results and Conclusion

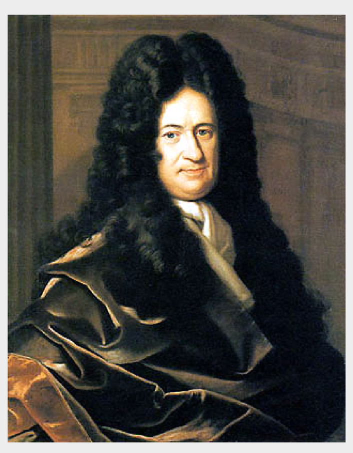
As it turns out, the ambiguity of natural language has given us room for two different formalizations of the same argument. Each of them allowed us to consider the subject from a different perspective, where previously unnoticed aspects have come into the foreground. The first variant being the essentialist nature of the concreteness predicate, and the second being the very idiosyncratic meaning given by the author to the terms necessity and contingency inside this argument. Both variants called our attention to several superfluous definitions and premises (not shown here) which, although used in the original natural language variant of the proof, were not necessary for the highly optimized proofs found by automated tools.

The formalization of philosophical arguments can play an important role in understanding the conceptual framework presupposed in philosophical theories, highlighting possible inconsistencies, redundancies and concepts in need of explanation. We want to present this analysis as an example of the adoption of formal methods in the philosophical discussion, and especially of the application of Automated Theorem Proving for the purposes of interpretation, simplification and scrutiny of metaphysical theories.

References

[1] Christoph Benzmüller and Lawrence Paulson. Quantified multimodal logics in simple type theory. *Logica Universalis (Special Issue on Multimodal Logics)*, 7(1):7–20, 2013.

[2] James Porter Moreland, Khaldoun A Sweis, and Chad V Meister. *Debating Christian Theism*. Oxford University Press, 2013.



Leibniz: *Calculemus!*

Computational Metaphysics is a interdisciplinary lecture course designed for advanced students of computer science, mathematics and philosophy. The main objective of the course is to teach the students how modern proof assistants based on expressive higher-order logic support the formal analysis of rational arguments in philosophy (and beyond). In our first course in Summer 2016 the focus has been on ontological arguments for the existence of God. However, some students picked formalization projects also from other areas (including maths).

Computational Metaphysics was awarded the *Central Teaching Award 2015* of the FU Berlin.