Formalization and Assessment of Lowe’s Modal Ontological Argument

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Project of Computational Metaphysics 2016
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Introduction
We present a formalization of a modal variant of the Ontological Argument put forward by E. J. Lowe [2]. The formalization of this argument has been carried out in an interactive theorem prover (Isabelle/HOL) using two different approaches. Firstly, a modal variant using a modified version of the Quantified Multimodal Logic (QML) embedding [1], and secondly, a simplified variant in classical predicate logic. Our central motivation has been to demonstrate the use of Automated Theorem Proving for clarifying concepts and verifying the validity of complex arguments in philosophy, especially metaphysics, and thus overcoming the limitations of intuitive natural language argumentation.

The Argument
The author (E. J. Lowe) introduces in his article a new modal variant of the Ontological Argument using a classical approach to philosophical argumentation; stating definitions and premises in natural language and then intuitively deducing further statements until finally obtaining the desired conclusion:

P1 God is, by definition, a necessary concrete being.
P2 Some necessary abstract beings exist.
P3 All abstract beings are dependent beings.
P4 All dependent beings depend for their existence on independent beings.
P5 No contingent being can explain the existence of a necessary being.
P6 The existence of any dependent being needs to be explained.
P7 Dependent beings of any kind cannot explain their own existence.
P8 The existence of dependent beings can only be explained by beings on which they depend for their existence.
C10 A necessary concrete being exists.

Modal Variant and Actualism
Two main historical developments motivate our modeling choice: (1) There is a long philosophical discussion, which dates back to ancient Greece, about the possibility of speaking about non-existent objects. (2) There is a belief that, in order to avoid the conclusion of the Ontological Argument, one must deny that existence is a property that can be predicated of objects. In this approach we want to circumvent both debates, by introducing actualist quantification to our modal formalization of the Ontological Argument. In a possible world semantics, this amounts to allowing universal and existential quantifiers to range over different domains at distinct possible worlds.

Formalization of Lowe’s Ontological Argument in QML
The main conclusion, stated in natural language, allows for ambiguous interpretations. We formalize and prove some of them:

FOL Variant
In the formalization above, we found ourselves eagerly interpreting the argument in a modal context for the sake of honoring the original intention of the author. However, a more literal reading promptly suggests another logical form. For instance, according to the author “there is no logical restriction on combinations of the properties involved in the concrete/abstract and the necessary/contingent distinctions. In principle, then, we can have contingent concrete beings, contingent abstract beings, necessary concrete beings, and necessary abstract beings.” By taking these four categories as exhaustive of our domain of discourse, a different reading of necessity and contingency reveals itself, not as modals, but as mutually exclusive predicates.

As a consequence, our universe of discourse (and some exemplary members) would look as follows:

<table>
<thead>
<tr>
<th>Necessary</th>
<th>Numbers</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>Fiction</td>
<td>Stuff</td>
</tr>
<tr>
<td>God</td>
<td>Contingent</td>
<td>Necessary</td>
</tr>
</tbody>
</table>

Consequently, the argument can be completely formalized in predicate logic and proved directly in Isabelle/HOL.

References

Figure 1: Embedding of quantifiers in HOL.

Figure 2: Embedding of modal operators in HOL.

Figure 3: Formalization of Lowe’s Ontological Argument in QML.

Figure 4: Embedding of existential quantifiers to range over different domains.

Figure 5: Figure 5: Non-modal Ontological Argument formalized in FOL.

Results and Conclusion
As it turns out, the ambiguity of natural language has given us room for two different formalizations of the same argument. Each of them allowed us to consider the subject from a different perspective, where previously unnoticed aspects have come into the foreground. The first variant being the essentialist nature of the concreteness predicate, and the second being the very idiosyncratic meaning given by the author to the terms necessity and contingency inside this argument. Both variants called our attention to several superficial definitions and premises (not shown here) which, although used in the original natural language variant of the proof, were not necessary for the highly optimized proofs found by automated tools.

The formalization of philosophical arguments can play an important role in understanding the conceptual framework presupposed in philosophical theories, highlighting possible inconsistencies, redundancies and concepts in need of explanation. We want to present this analysis as an example of the adoption of formal methods in the philosophical discussion, and especially of the application of Automated Theorem Proving for the purposes of interpretation, simplification and scrutiny of metaphysical theories.