Proving God’s Existence by Automating Leibniz’ Algebra of Concepts

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Introduction
From 1679 onwards Leibniz developed a modal Logic of Concepts. Concepts play the role of predicates in ordinary predicate logic. Wolfgang Lenzen showed [2] that one can easily apply Leibniz own logic to formalise a famous ontological argument by Leibniz in his own logic. From this starting point we formulated Lenzen formalisation in the proof assistant Isabelle/HOL and conducted (semi-) empiric experiments using Isabelle’s automation tools.

Leibniz’ Axioms
Leibniz defines three primitive operations on concepts.

- Concept Containment: ∈
  “Blue” contains “colored”
- Concept Conjunction: +
  Combines two concepts into a composite concept
- Concept Negation: ¬
  Returns the “opposite concept”

The embedding of concepts, its primitive and definable operations, and Lenzen’s axiomatization in Isabelle can be seen in Figure 1.

Preliminaries to the Divine
Leibniz’ ontological argument deals with necessary, possible and existence. These three terms have to be defined before we start the proof itself.

- Existence is just a special concept “E”.
  From today’s perspective this seems inappropriate.
- Possibility “P” is a derived notion; it attaches to concepts not propositions.
  \[ P(C) : \forall A : (C \notin \{A, \sim A\}) \]
- Leibniz proposed several notions of necessity over the course of his life. We use the straightforward interpretation here.
  \[ N(C) : \forall P \sim P(C) \]
- Counterintuitively, Leibniz’ modal logic is extensional.

Proving God
Having defined the logic and basic concepts we can now state what it means for god to be necessary.

- The term “being god(ly)” is identified with a concept G.
- The desired conclusion “god exists” is identified with \( G \in E \) – the concept of god is contained in the concept of existence.
- Interestingly (see the paragraph below) the only working axiomatization for god as the necessary being seems to be \( N(\sim G \lor E) \) or with “Concept Implication” \( N(G \rightarrow E) \) not \( N(G) \)

Results and Conclusion
In our work we were able to

- confirm that Leibniz’ axiom system is consistent.
- computationally verify and improve upon Lenzen’s formalisation.
- easily prove some worrying statements using Leibniz’ system (e.g. Whatever possibly exists, exists actually).

We also found novel and perhaps philosophically interesting facts about Leibniz’ ontological argument.

Leibniz uses “ens necessarium” and “ens ex cuius essentia sequitur existentia” interchangeably. In his own system however, there is profound difference between \( N(G) \) and \( N(G \rightarrow E) \). If we use the former, the proof fails and Isabelle’s nitpick routine quickly finds a countermodel. The latter works as advertised. Our results, especially the countermodels, will be published soon as a book chapter [1].

References

Figure 1: Axiomatization in Isabelle/HOL

Figure 2: The entire proof in Isabelle/HOL