Higher-Order Metaphysics

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Introduction

Some Examples

Two Applications Gödel Might Have Found Interesting

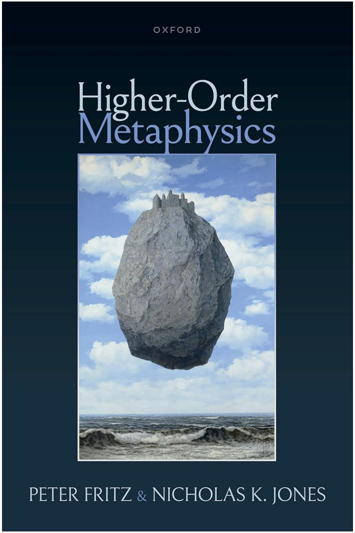
The Modal Ontological Argument

The Continuum Hypothesis

What is higher-order metaphysics?

> *"Higher-order metaphysics uses the formal languages of higher-order logic to formulate metaphysical views and arguments."*
>
> *(Fritz and Jones, 2024)*

Recent Developments



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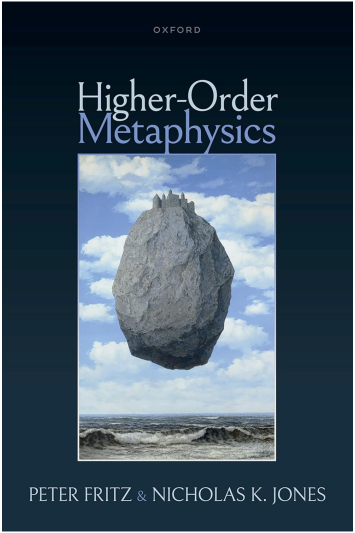

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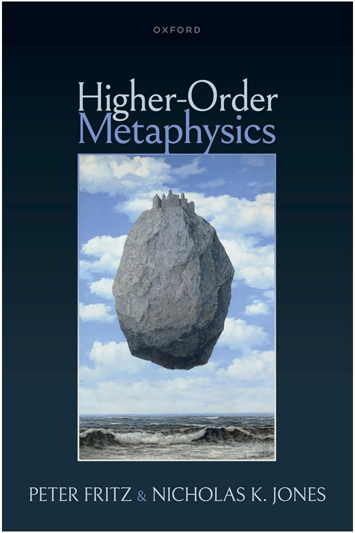

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- Formal and natural languages need not correspond exactly.
- The intended interpretation of the formal language need not be provided by a proof theory or model theory.
- Classical principles of higher-order quantification, and at least extensional β-conversion:

$$(\lambda x.\varphi)t \leftrightarrow \varphi[t/x]$$

Breaking the Domination of the Word over the Human Spirit

This is all not far from a suggestion in Frege's *Begriffsschrift*:

> *"If it is one of the tasks of philosophy to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of language often almost unavoidably arise concerning the relations between concepts and by freeing thought from that with which only the means of expression of ordinary language, constituted as they are, saddle it, then my ideography, further developed for these purposes, can become a useful tool for the philosopher."*
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> *(van Heijenoort, 1967, p. 7)*

Necessitism

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Williamson holds that these principles articulate interesting metaphysical views, which can be expressed in purely logical terms.

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$$mp = nq \to m = n \wedge p = q$$

(The argument goes back to Russell (1903) and Myhill (1958).
It can be adapted to an argument against grounding (Fritz, 2022).)

Classicism

Recently, there has been a lot of interest in *Classicism*, a view which can be articulated by adding to classical higher-order logic the following rule of inference:

$$\text{From } \varphi \leftrightarrow \psi \text{, derive } (\lambda x_1 \ldots x_n . \varphi) = (\lambda x_1 \ldots x_n . \psi)$$

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- I argue this is *metaphysical necessity* (Fritz, 2023), given what Kripke (1980 [1972]) says; others disagree (Bacon, 2018).
- Classicism entails that $\Box$ (as defined) satisfies S4.

S5 and Possible Worlds

Assuming *Classicism*, the following are equivalent:

$$(5)\quad \Diamond p \to \Box\Diamond p$$

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(Similar arguments can be found in Gallin (1975), Fine (1980) and Menzel and Zalta (2014).)

Weaker Modal Logics

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Not all agree that it includes 5, and so S5:

- Bacon (2018, 2020) appeals to vagueness and combinatorial principles to argue against 5.
- Ditter (2020, 2022) reduces necessity to essence and argues that this requires rejecting 5.
- Roberts (2023) argues against ND_e (i.e., $x^e \neq y^e \to \Box(x \neq y)$), which requires rejecting 5.

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- This could help establish confidence in the claimed result.
- It could also help establish new results.

The Modal Ontological Argument (MOA)

(The following is based on Fritz et al. (forthcoming).)

A simple modal version of Anselm's ontological argument:

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The argument is valid in, e.g., S5.
The premises are naturally motivated as follows:

(P1) The be God is to be perfect, i.e., to have every perfection. Necessary existence is a perfection.

(P2) God's existence is conceivable, and therefore possible.

The Reverse Modal Ontological Argument (RMOA)

The argument has a natural analog in favor of *atheism*:

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The argument is also valid in S5.
And the same reasoning motivates the premises:

(P2*) God's non-existence is conceivable, and therefore possible.

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  Q1: Are there any independent reasons for adopting a modal logic sufficiently weak for this?

  Q2: What does such a position entail about the existence of God?

A Sufficiently Weak Modal Logic

We require a modal logic in which P1, P2, and P2* are consistent. So, a logic not containing Inc:

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Returning to the MOA/RMOA

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  But the weak logic KT suffices to for the validity of RMOA.
- So if we accept P1, P2, and P2* (and accept KT), we must be atheists.
- Of course, the argument is only as convincing as the premises.
- However, there are more sophisticated defenses of the premises, e.g., in Gödel's own work.

The Continuum Hypothesis (CH)

(The following is based on Fritz (unpublished).)

Background:

- CH: there is no cardinality between $|\mathbb{N}|$ and $|\mathbb{R}|$, i.e., $|\mathcal{P}(\mathbb{N})|$.
- CH is independent of ZFC (Gödel, 1940, Cohen, 1966).

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- CH is independent of ZFC (Gödel, 1940, Cohen, 1966).
- Gödel (1947) hoped to resolve CH by motivating further set-theoretic axioms.
- No such resolution has yet been accepted in mathematics.

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(1) The view is attractive, or at least not implausible.

(2) The view doesn't obviously prejudge controversial questions in (the philosophy of) set theory.

(3) Nevertheless, the view settles the continuum hypothesis.

The last claim is established using a deduction in classical higher-order logic.

A Sketch of the View

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- Assume there are at least $\aleph_2$ many logical atoms.

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- I conjecture that the view is consistent, but I don't have a worked-out proof.
- Interestingly, it becomes inconsistent when we assume that there are $\beth_2$ logical atoms. This is concerning for state-space theories.
- The argument isn't meant to establish $\neg$CH, since there are likely equally plausible views which entail CH.

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