A Stability Interpretation of Gödel

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Introduction

- Gödel held a puzzling collection of views:
  1. Realism about mathematics
  2. Idealism about time
  3. Ontological proof
- Question: Is there an epistemological principle which renders these view coherent?
  - Systematically rather than historically
  - epistemically rather than metaphysically
- Answer: The principle of stability.

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Scientific Modeling and Stability

Vindicating Gödel's Views

Evaluation and Change of Context

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Scientific modeling

- Phenomenon: that is studied by the scientist
  - Example: Universe at large
- Property: of the phenomenon and the scientist would like to know if it obtains
  - There is a 'global time' (that all observers agree on)
- Theory: laws governing the phenomenon, in a language that can express the property
  - General relativity theory
- Models (of the theory): mathematical structures representing ways the phenomenon could be given the laws (i.e., epistemically possible worlds)
  - Spacetimes (certain 4-dimensional manifolds)

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Principle of stability I

- When can we infer that the phenomenon has the property, given a model with the property?
- Principle of stability: Only if the model has the property stably, i.e., all relevantly similar models have the property, too [Fle20].
- Example:
  - Say our scientists have evidence that spacetime M represents the actual universe.
  - They mathematically prove M has a global time (φ).
  - The evidence is subject to measurement errors, etc.
  - Conclude φ for the universe only if all models up to measurement error have φ.

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Principle of stability II

Scientific modeling context := phenomenon + property + theory + models + similarity

Principle of stability (necessity)

In a scientific modeling context, the inference from a model having property φ to the phenomenon having property φ is justified only if the model has property φ stably, i.e., all relevantly similar models also have φ.

Principle of stability (sufficiency)

In a scientific modeling context, if all models have property φ ('maximal stability'), then the inference from a (or any) model having property φ to the phenomenon having property φ is justified.

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Modal argument pattern for Gödel's views

(P1) Identify the scientific modeling context.

(P2) Prove the non-stability (resp., maximal stability) of the property φ in the class of models.

(C) Conclude with the stability principle that φ cannot (resp., can) be justifiably inferred about the phenomenon.

Goal:

- Time: context with no justification for global time.
- Mathematics: context with justification of some mathematical objects.
- Theism: context with justification of God.

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Idealism about time

(P1) Scientific modeling context:

- Phenomenon: universe at large
- Property: global time[1]
- Theory: general relativity
- Models: spacetimes
- Similarity: maximal (any M is similar to any M')

(P2) Gödel spacetimes exist, which cannot have a global time.

(C) So 'having a global time' is not (nowhere) stable. Hence it cannot be justifiably concluded about our universe.

[1] There is $t : M \to \mathbb{R}$ such that, for all events $p, q \in M$, if p causally precedes q iff $t(p) \leq t(q)$.

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Realism about mathematics I

(P1) Scientific modeling context:

- Phenomenon: "mathematics"
- Property: "existence of mathematical objects", say the set of natural numbers.
- Theory: ZFC
- Models: (transitive) models (M, E) of ZFC
- Similarity: maximal similarity

(P2) Absoluteness theorems in set theory: The set of natural numbers $\mathbb{N}$ of our background theory is identical to the set of natural numbers $\mathbb{N}^M$ of any transitive model M of ZFC.

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Realism about mathematics II

- This way other absolute objects/concepts can be justified: being a pair, function, ordinal, etc.
- But two ways this can fail for non-absolute objects/concepts:
  (1) There is no formula that uniquely defines the object/concept (in our background theory).
  (2) There is a formula, but it changes its meaning across models.
- Example of (2): The continuum $2^{\mathbb{N}}$ (due to Cohen)
- Example of (1): the "Absolute", an example Gödel was excited about …

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Realism about mathematics III

[Gödel] brought up ... one of his favorite concepts, the Absolute, with a capital 'A'. That's not a concept that 20th century philosophers tend to bring up: the Absolute. So what was it to him? In my mind, it was the class of all sets, but maybe it was something more for him, I'm not sure. But he seemed to be talking about the class of all sets, hence of all mathematical objects; that's worthy of being called the Absolute. ... He says: 'You know, language does not enable us to define the Absolute'. If you have a formula, $F(x)$, it can never happen that that formula has exactly one x that satisfies it, that x being the Absolute, that can't happen. Because we know from the reflection principle in set theory, if I have a formula $F(x)$, and $F(V)$ holds where V is the class of all sets, then F must also hold for some set. In other words, anything you say about the class of all sets which is true, is also true for some particular set. And hence you can't define the Absolute. ... Then his eyes lit up, he said: 'Isn't that wonderful? We know something important about the Absolute, just by logic, just by reason'. And he was full of enthusiasm.

[Gerald Sacks, https://www.youtube.com/watch?v=PR7MTqtFl4Y (25:39)]

Ontological proof

(P1) Scientific modeling context:

- Phenomenon: theism
- Property: existence of God
- Theory: Gödel's theory of an object which has all the positive properties
- Models: models of higher-order modal logic satisfying Gödel's theory
- Similarity: maximal similarity

(P2) Gödel shows that the theory implies the existence of God, so this holds in all models; formally verified by [BW14].

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Now we can evaluate these argument by assessing/changing the scientific modeling context.

1. Change theory: e.g., Gödel's program of finding new axioms in set theory (to settle continuum hypothesis)
2. Change property: e.g., other 'modal arguments' in general relativity, namely validity of Church–Turing thesis (Malament–Hogarth spacetimes)
3. Change similarity: more refined notion of similarity (not just maximal similarity).

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Explicating similarity via topology

If X is a class of models, explicate (graded) similarity via topology [Fle20; Fle18b]:

- Given a model x in X, degree of similarity to x = the set U of models that are similar to x to that degree.
- $\mathcal{N}_x$ = the collection of x's similarity degrees.
  - $\forall U \in \mathcal{N}_x : x \in U$
  - $\forall U, U' \in \mathcal{N}_x \; \exists V \in \mathcal{N}_x : V \subseteq U \cap U'$
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Stability topologically

- A property P of models is a subset of the class X of models.
- A model $x \in X$ has property P stably if

$$\exists U \in \tau : x \in U \subseteq P.$$

- Property $P \subseteq X$ is stable if any model $x \in X$ that has P has it stably.

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Gödel's argument reconsidered

- Gödel's stability requirement criticized as too strong.
- Choosing 'correct' stability now turns into choosing 'correct' topology on class X of spacetimes.
- Debated in 1970s: [Ger70; Ger71; Haw71; Ler73]
- Two main contenders:
  - In open topology, having global time is stable [Haw69]. So Gödel's argument does not succeed.
  - In compact-open topology, failing to have a global time is dense [Fle16]. So Gödel's argument succeeds.
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- Modal logic ($\Box\varphi$ as 'stably φ')
- Stone duality (more in the paper).

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Outlook II

- Choosing the ‘right’ topology as second-order stability
  - Each topology τ is one model of the meta-phenomenon ‘similarity’.
  - Given the original property φ (e.g., global time), consider the meta-property

$$\psi(\tau) := \text{property } \varphi \text{ is stable in topology } \tau.$$

  - Second-order stability: only if ψ is itself stable at τ, we can infer ψ for τ!
- Which topological features persist when varying the topology is studied in persistent homology (in topological data analysis) [Wei11; EH08; Ott+17].

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- For each view, we provided a scientific modeling context in which the principle of stability entails the view held by Gödel.
- Changing context: add new axioms, apply to other properties, or refine notion of similarity.
- Explicate similarity as topology, then Gödel's idealism becomes sensitive to choice of topology.
- Second-order stability as interesting research direction.

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- Explicate similarity as topology, then Gödel's idealism becomes sensitive to choice of topology.
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