Freie Universität Berlin

Special and General Relativity Theory

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The Special Theory of Relativity

Classical Mechanics and the restricted Principle of Relativity

- An inertial reference frame is called a "Galilean system of co-ordinates“. [1]

- If K is a Galilean coordinate system, every coordinate system K’ which is, in relation to K, in a condition of uniform motion of translation, is a Galilean one and the laws of Newton hold for K’ as they do for K.

- The theorem for addition of velocities from Newton mechanics is in contradiction with the constant speed of light in vacuum. [1][2]

The Special Theory of Relativity

- “The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform motion of translation”.[3]

- The speed of light in a vacuum is a constant $c = 299\ 792.458\, km/s$.

- No speed can be higher than speed of light. [4][5]

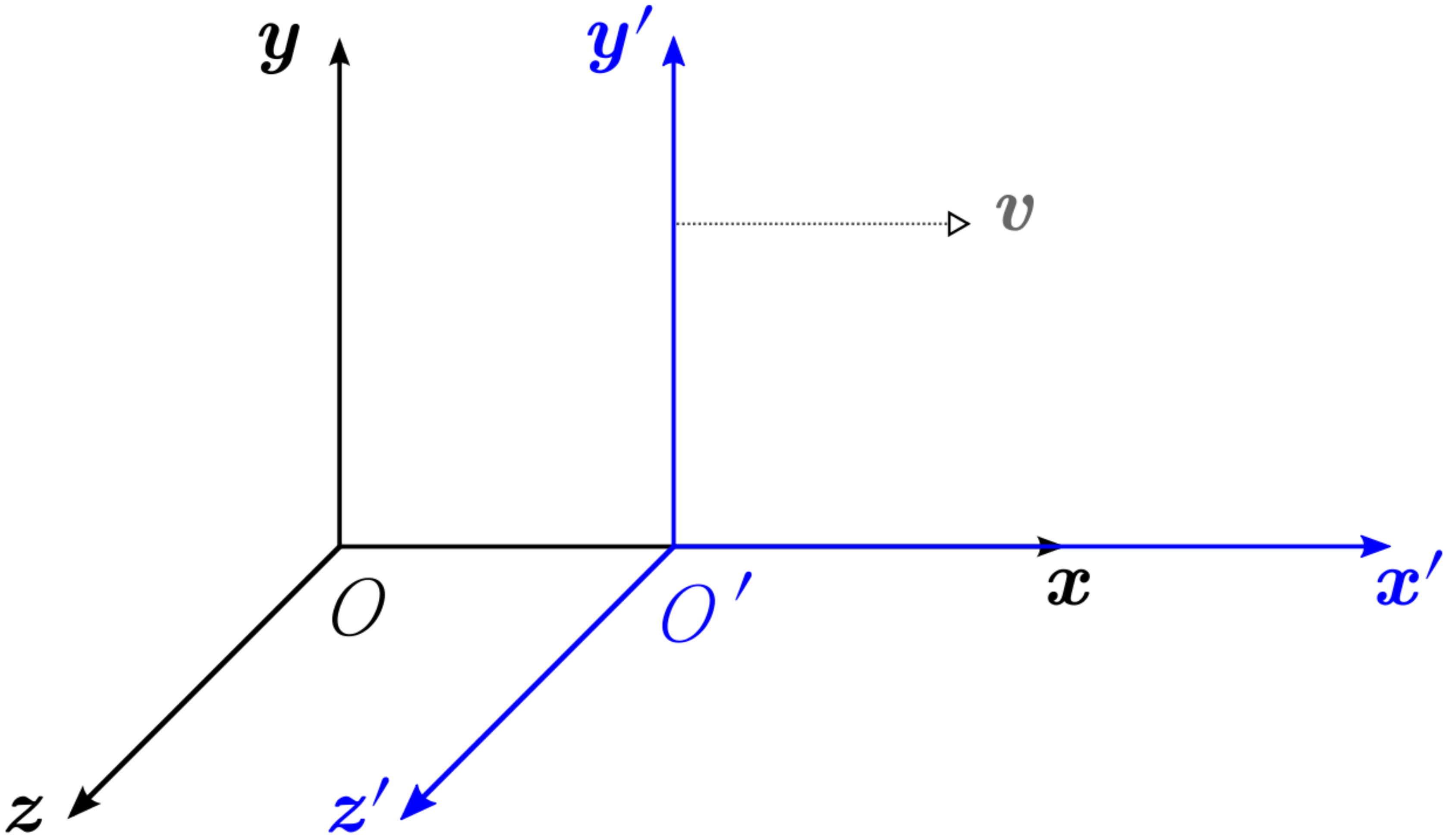
- An infinite amount of energy is needed in order for a particle with a mass m to reach the speed of light c. [4]

$$E = \frac{\mathrm{m}\ c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Lorentz Transformation

- According to classical mechanics: The time-interval (time) and the space-interval (distance) are independent of the motion of the body of reference. [1]

- According to Special Theory of Relativity, the values x', y', z', t' of an event in coordinate system K', which is moving with velocity v in reference to the X-axis of the K coordinate system can be calculated from the values x, y, z, t of the same event in reference to K with the Lorentz Transformation such as the speed of light remains constant.



Source: https://commons.wikimedia.org/wiki/File:Frames_of_reference_in_relative_motion.svg

- **Lorentz Transformation**(for movement in direction of x-axes)

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y‘=y$$

$$z‘=z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- **Galilei Transformation**(for movement in direction of x-axes)

$$x' = x - vt$$

$$y‘=y$$

$$z‘=z$$

$$t‘=t$$

- Each individual event by the four dimensions of Minkowski’s space-time, has three space coordinates x, y, z, and a time coordinate, t.

- Every coordinate system has its own particular time. [1]

- Simultaneity for an observer A can be different for another observer B who is in a relative motion in reference to observer A.

- Effect of time dilation: Slower ticking of moving clock measured in comparison to a stationary clock.

- Measured distance is also dependent of the coordinate system.

- Effect of length contraction: Shortening in direction of moving in reference to observer.

- As an example for time dilation can be given the case of numbers of muons reaching the Earth.

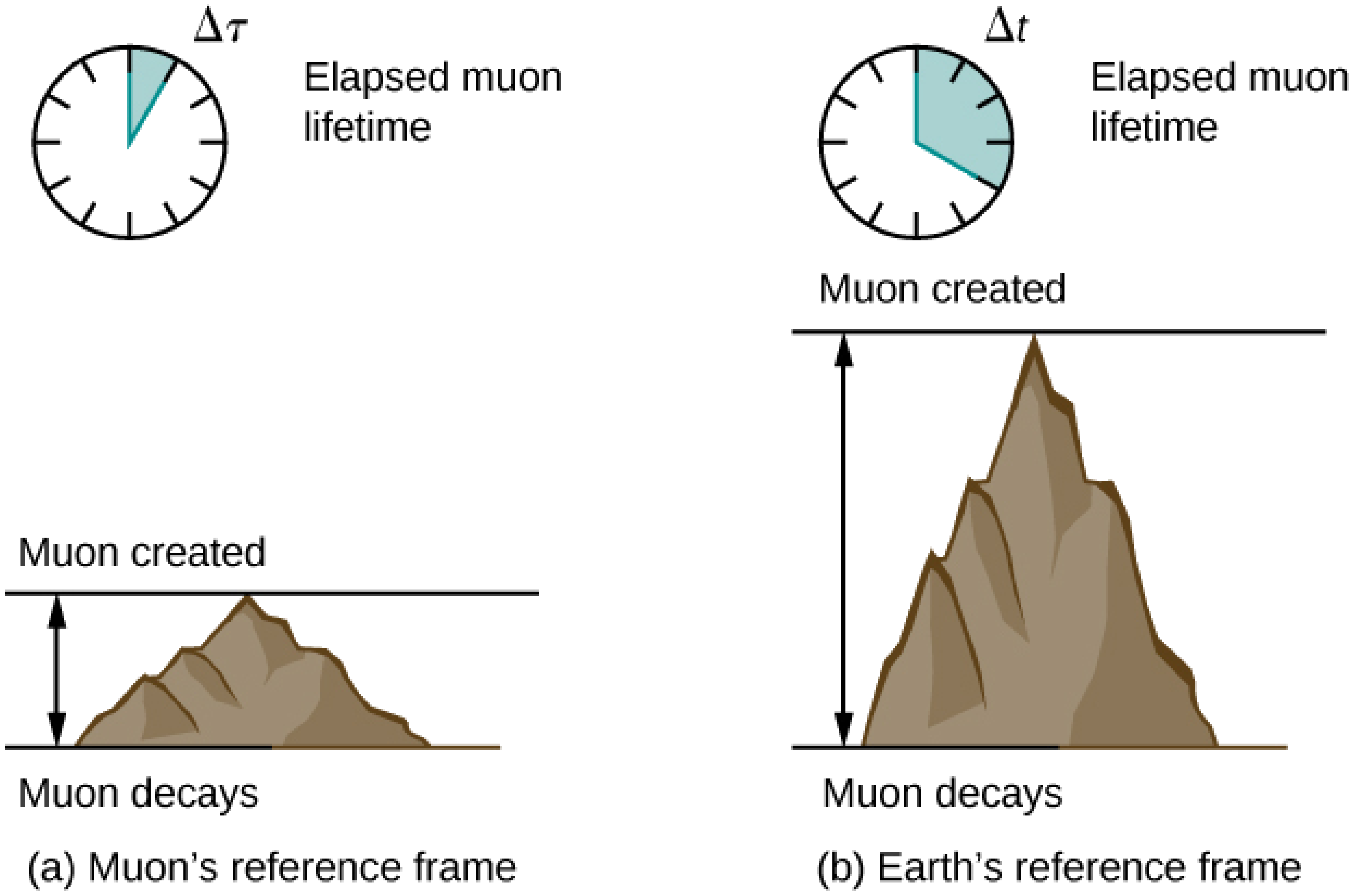
- Muons are subatomic, unstable particles that are released by cosmic radiation at a height of approximately 10 km above the earth’s surface.[6]

- Their speed is 99.5% of that of the speed of light c. In their reference frame, they can cover only $450\mathrm{m}$ in the course of half of their lifespan, $\tau_0 = 1.5\mu s$. Thus the probability that muons would reach Earth is very low.

- However, due to their speed in reference to the Earth, their lifespan in Earth's reference frame is $\tau\ = 15\mu s$.

$$15\mu s = \frac{1.5\mu s}{\sqrt{1 - \frac{(0.995)^2 c^2}{c^2}}}$$

- The measurements prove that muons reach the surface of Earth and this proves the existence of time dilation and length contraction.[7]



Source: https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/5%3A__Relativity/5.3%3A_Time_Dilation

The General Theory of Relativity

Newton's law of universal gravitation

- Each body attracts every other body in the universe with a force which is proportional to their masses and inversely proportional to their distance. [1]

$$F = \frac{m_1 m_2}{r^2} G$$

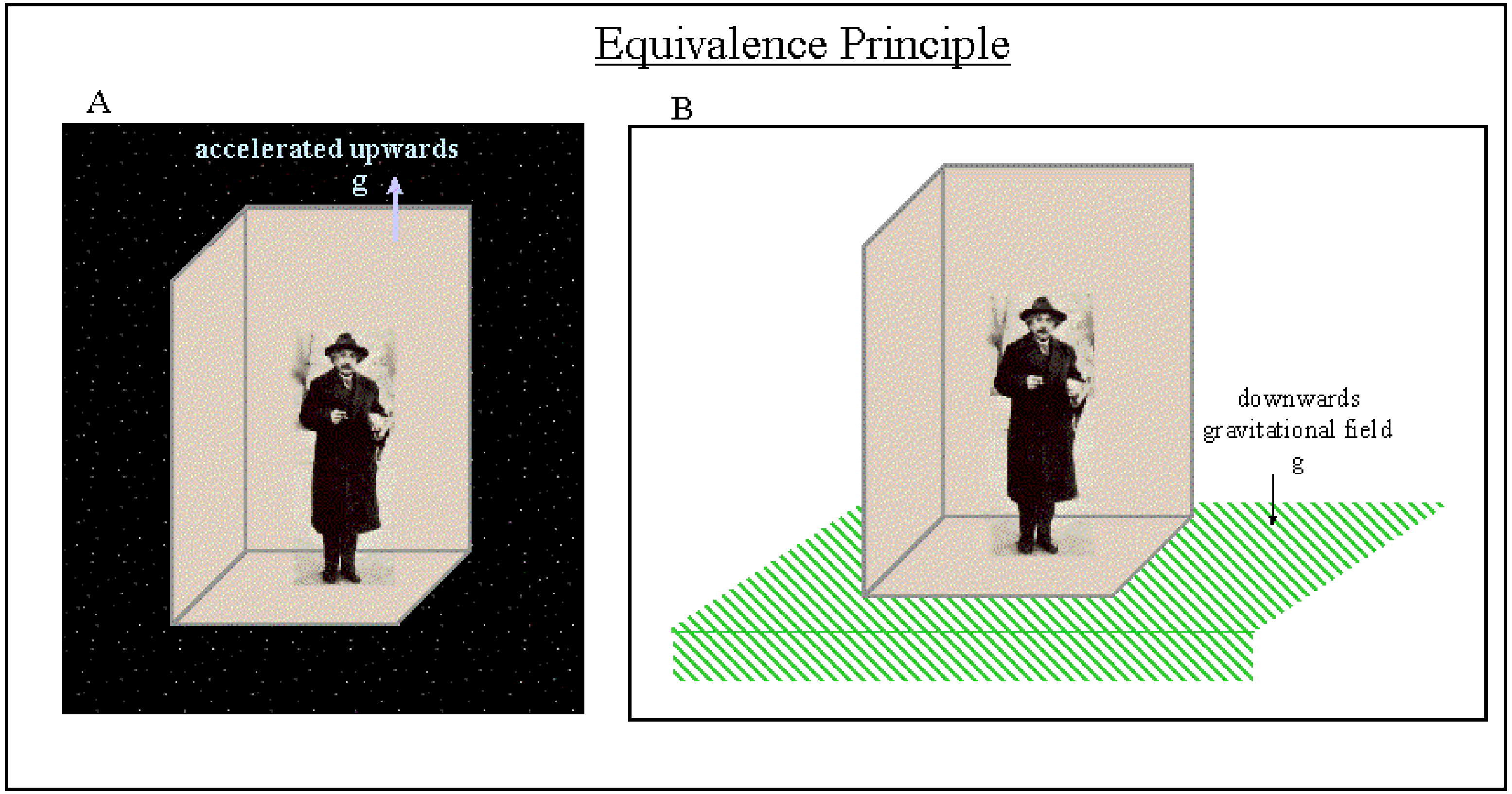
- $m_{1,}\, m_2$ are the masses of the bodies.
- G is the Newton gravitational constant.
- r is the distance between the bodies.

Formulation of the General Principle of Relativity

- All reference frames are equivalent for the formulation of the general laws of physics. [1]

- Equivalence principle requires that all reference-frames can be used as reference-frames with equal right and equal success in the formulation of the general laws of nature.

- General theory of relativity states that by application of arbitrary substitutions of the coordinate variables the physical equations must pass over into equations of the same form for each reference frame.



Source: http://astrosun2.astro.cornell.edu/academics/courses/astro201/equivalence_princ.htm

- According to general theory of relativity, every transformation (not only the Lorentz transformation) corresponds to the transition of one coordinate system into another.

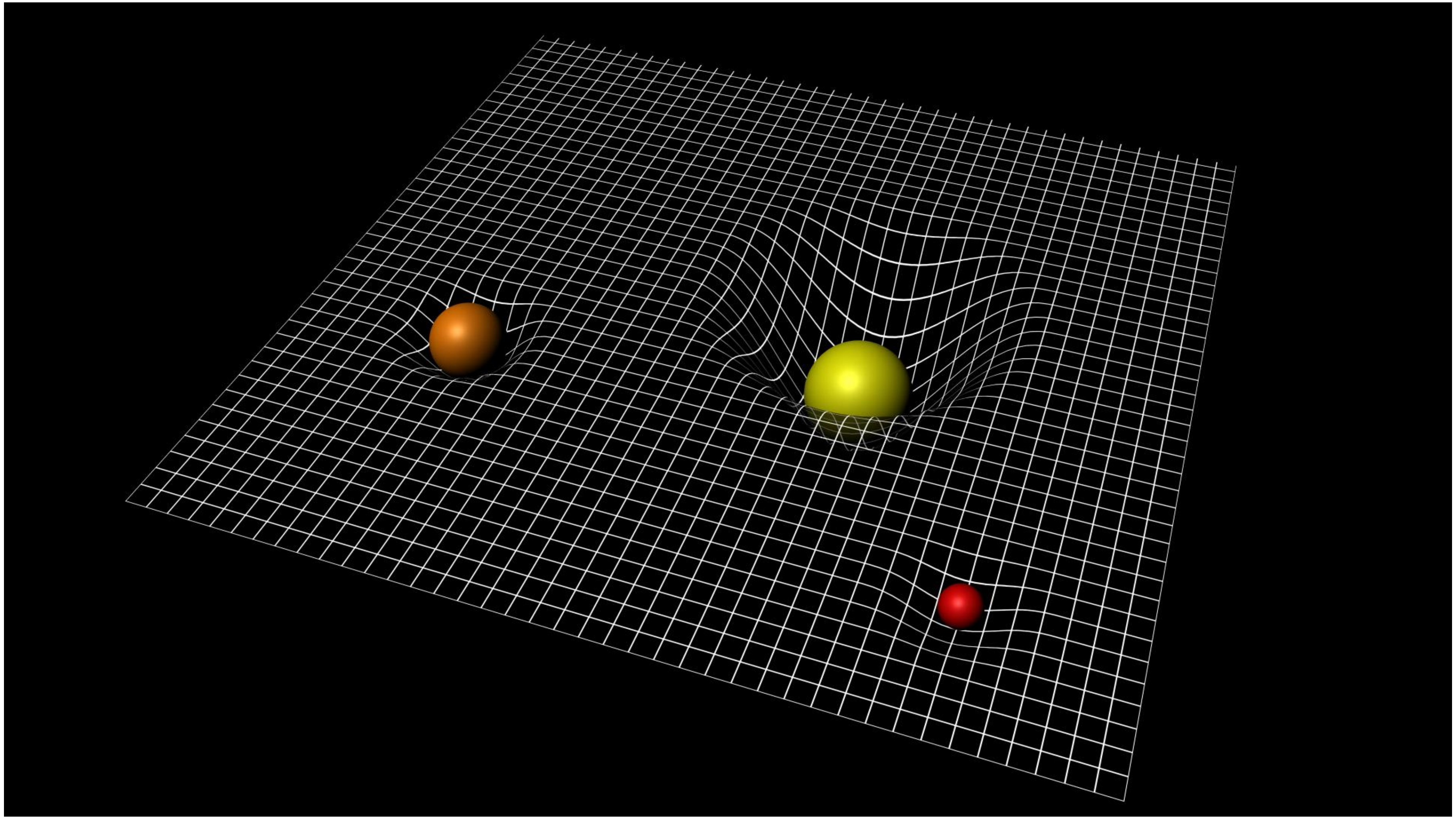
- No difference in free fall in a gravitational field or being idle in a falling elevator.

- Gravitation can be simplified as a curvature of spacetime, caused by presence of energy and matter.

- Curvature of spacetime dependent of the mass and energy present. G on stands for the curvature of spacetime at a certain point and T is the mass at that point and its properties: [8]

$$\boldsymbol{G}_{\mu v} + \Lambda \boldsymbol{g}_{\mu v} = \frac{8\pi G}{c^{\Lambda}} \boldsymbol{T}_{\mu v}$$

- $G_{\mu v}$ is the Einstein tensor which represents the curvature of spacetime in a certain point.
- Λ represents the cosmological constant.
- $g_{\mu v}$ is the metric tensor which represents the geometrical structure of spacetime.
- G is the Newton gravitational constant.
- $T_{\mu v}$ is the stress-energy tensor and it represents the flux and the density of energy and momentum in a certain point in spacetime.
- c is the speed of light.



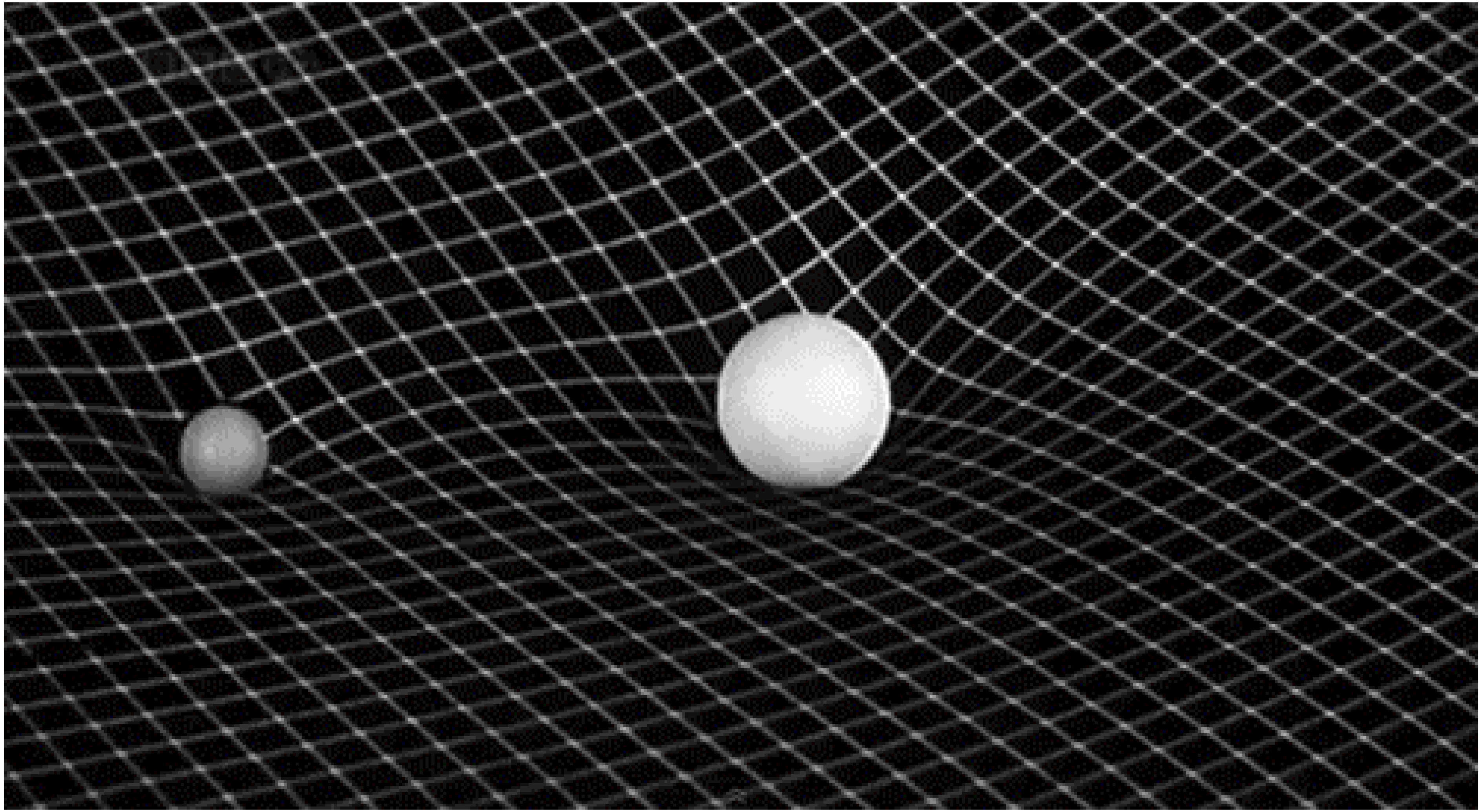
Source: https://www.esa.int/spaceinimages/Images/2015/09/Spacetime_curvature

- Effect of gravitational time dilation. Slower ticking of clocks due to gravitational field.

- Effect of light bending. Light rays are bent in the presence of gravitational field.

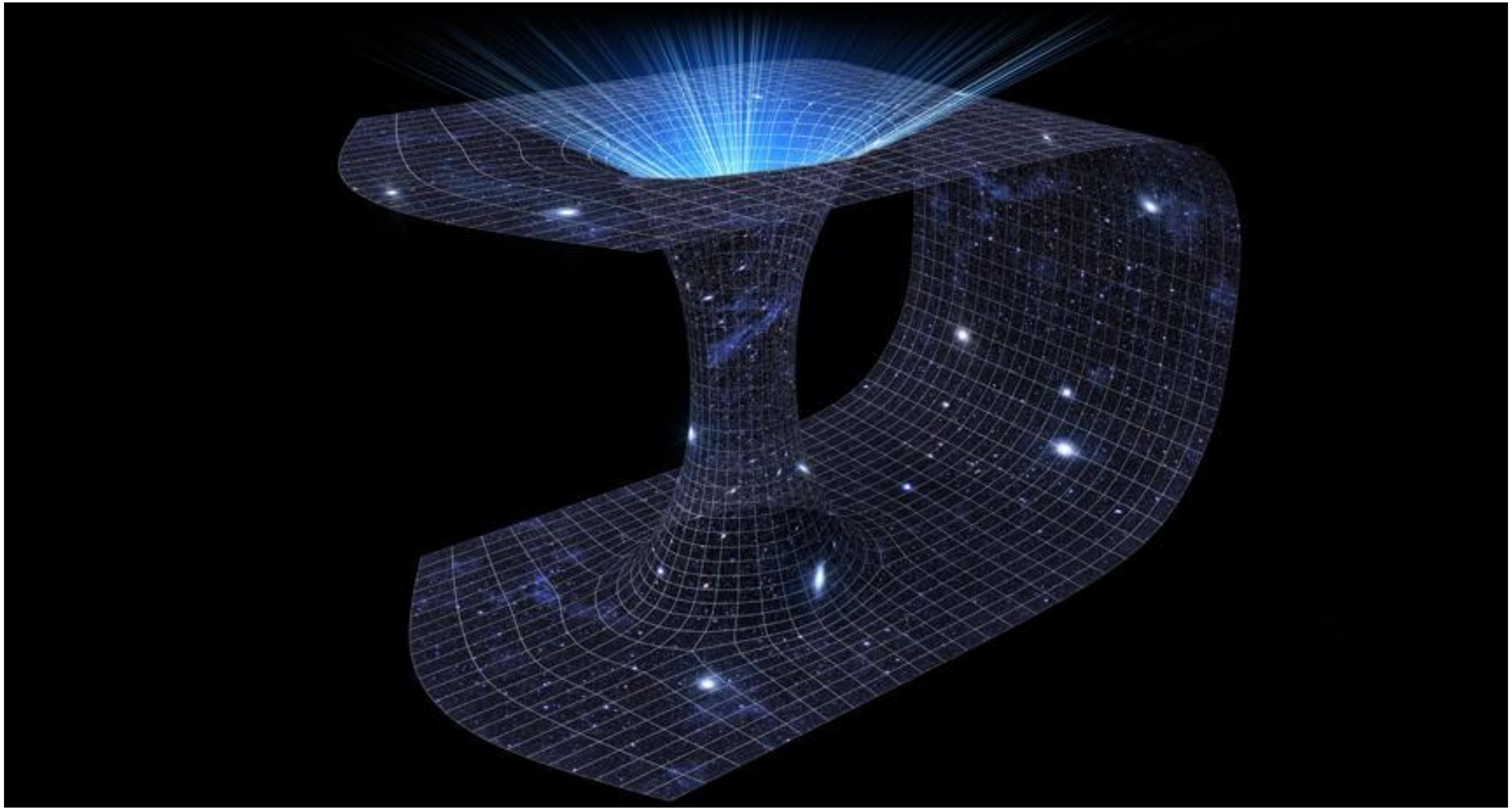
- Intuitively, gravitational field can be explained as curvature in the spacetime.

- Planets rotating around the sun are simply following the curvature of spacetime caused by Sun’s matter and energy.



Source: http://reekoscience.com/science-news/space/gravitational-waves-discovery-explanation-importance

- Theoretical possibility of time travel with spacetime bending.[9]
- By bending spacetime, a closed loop to the future might be formed.
- No travelling in the spacetime before the bending was made.
- However, that bending would require an unforeseeable amount of energy, close to the magnitude of a black hole.
- This time machine would be very unstable.



Source: https://www.sciencenews.org/blog/context/new-einstein-equation-wormholes-quantum-gravity

Thank you for your attention!

[1] Albert Einstein. (1916). *Relativity: The Special and General Theory*. H. Holt and Company.

[2] Albert A. Michelson, Edward W. Morley. (1887). *On the relative motion of Earth and the luminous effect*. *American Journal of Science*.

[3] Albert Einstein. (1905). *On the electrodynamics of moving bodies*.

[4] William Bertozzi. (1964). *Speed and Kinetic Energy of Relativistic Electrons*. American Journal of Physics.

[5] Shanchao Zhang, J. F. Chen, Chang Liu, M. M. T. Loy, G. K. L. Wong, Shengwang Du. (2011). *Optical Precursor of a Single Photon.* Physics Review Letters.

[6] Oliver Passon. *Kurzfassung der speziellen Relativitätstheorie*. http://www.psiquadrat.de/downloads/SR-Zusammenfassung-kurz.pdf

[7] D. H. Frisch, J. H. Smith.(1963). *Measurement of the Relativistic Time Dilation Using μ-Mesons*. American Journal of Physics.

[8] Øyvind Grøn, Sigbjørn Hervik. (Version 9th, 2004 - Draft). *Einstein's General Theory of Relativity.* ©Grøn & Hervik.

[9] Amos Ori. (2007). *Formation of closed time like curves in a composite vacuum/dust asymptotically-flat spacetime*. Physical Review Letters.