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Special and General Relativity Theory

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An inertial reference frame, one in which a body is idle or is moving at a constant linear velocity if not applied force, is called a "Galilean system of co-ordinates ". [1] If K is a Galilean coordinate system, every coordinate system K' which is, in relation to K, in a condition of uniform motion of translation, is a Galilean one and the laws of Newton hold for K' as they do for K. The theorem for addition of velocities from Newton mechanics is in contradiction with the constant speed of light in vacuum, which has been experimentally proven, thus opening the need for special theory of relativity. [1][2]

The Special Theory of Relativity

"The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform motion of translation".[3] The speed of light in a vacuum is experimentally proven to be a constant $c = 299\,792.458\, km/s$. The speed of light is the maximum velocity that can be reached. [4][5] However, it is reached by particles without a mass, for example photons which are massless particles. For a particle with a mass m, an infinite amount of energy is needed for a particle with a mass *m* to reach the speed of light *c*. Experimentally has been proven that no matter how much kinetic energy is added to particle with a mass *m*, its speed can't reach a velocity *c* [4]

$$E = \frac{\mathrm{m}\, c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Lorentz Transformation

According to classical mechanics the time-interval (time) and the space-interval (distance) are independent of the motion of the body of reference. [1] According to Special Theory of Relativity, by taking in account the constant speed of light and not permitting equal or higher velocities to the speed of light through adding of velocities, the values *x', y', z', t'* of an event in coordinate system K', which is moving with velocity *v* in reference to the X-axis of the K coordinate system can be calculated from the values *x, y, z, t* of the same event in reference to K with the Lorentz Transformation such as the speed of light remains constant. The term Minkowski space-time is introduced, so time becomes coordinate in the four-dimensional space-time. Each individual event by the four dimensions of Minkowski's space-time, has three space coordinates x, y, z, and a time coordinate, t.

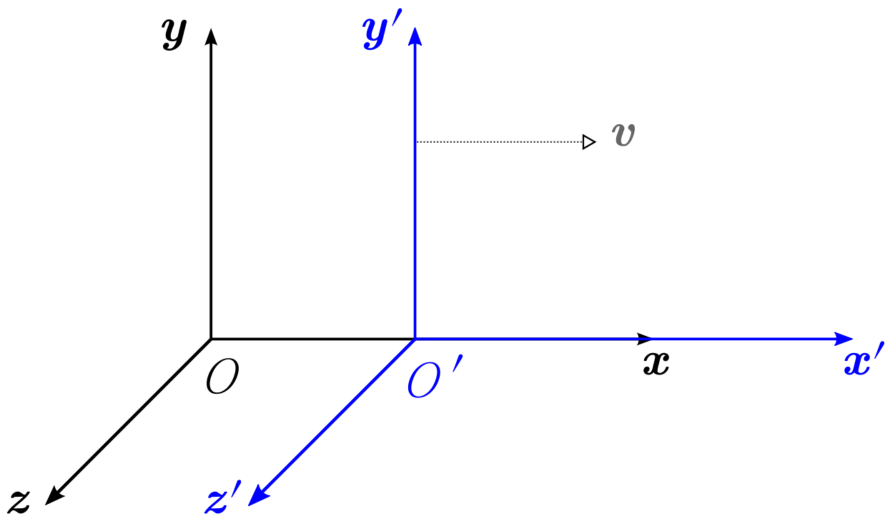

Figure 1. Reference frames in relative motion [1]

Lorentz Transformation (for movement in direction of x-axes)

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Galilei Transformation (for movement in direction of x-axes)

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

From the above stated can be concluded that every coordinate system has its own time. [1] Thus simultaneity for multiple observers can be different if one of the is moving with a velocity v in reference to the other one(s). By applying the Lorentz transformations, can be noticed the effect of time dilation and length contraction. Namely, the slower ticking of moving clock in comparison to a stationary clock and the dependence of the measured distance to the coordinate system in which is measured. Due to the length contraction, there is length shortening in direction of moving in reference to observer.

Muons as time dilation and length contraction example

As an example, for time dilation can be given the case of numbers of muons reaching the Earth. Muons are subatomic, unstable particles that are released by cosmic radiation at a height of approximately 10 km above the earth's surface.[6] Their speed is 99.5% of that of the speed of light c. In their reference frame, they can cover only 450m in the course of half of their lifespan, $\tau_0 = 1.5\mu s$. Thus the probability that muons would reach Earth is very low. However, due to their speed in reference to the Earth, their lifespan in Earth's reference frame is $\tau = 15\mu s$.

$$15\mu s = \frac{1.5\mu s}{\sqrt{1 - \frac{(0.995)^2 c^2}{c^2}}}$$

The measurements prove that muons reach the surface of Earth and this proves the existence of time dilation and length contraction.[7]

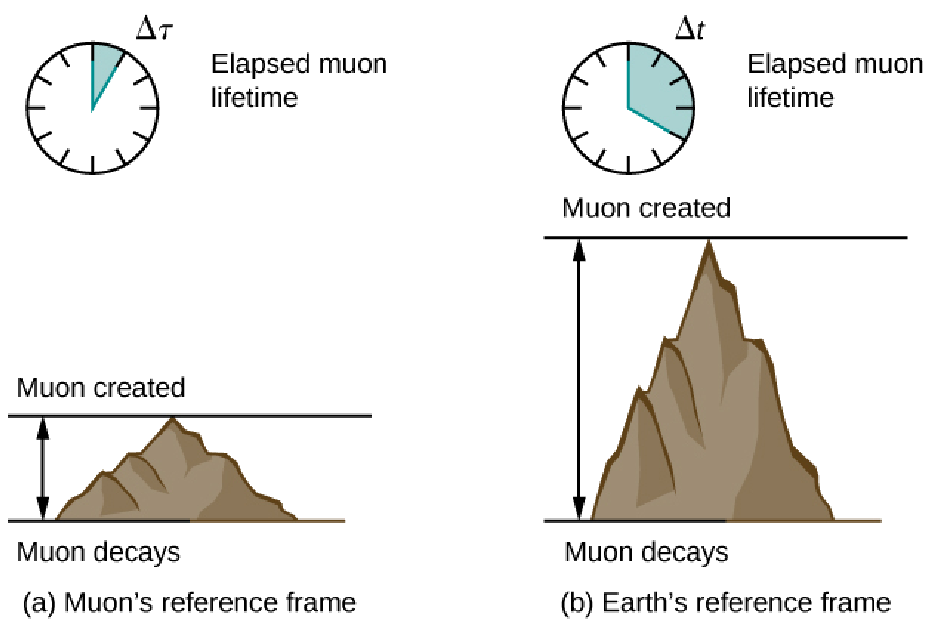

Figure 2. Time dilation and length contraction in muons [2]

The General Theory of Relativity

Newton's law of universal gravitation

Each body attracts every other body in the universe with a force which is proportional to their masses and inversely proportional to their distance. [1]

$$F = \frac{m_1 m_2}{r^2} G$$

- $m_{1,}\, m_2$ are the masses of the bodies.
- G is the Newton gravitational constant.
- r is the distance between the bodies.

Formulation of the General Principle of Relativity

All reference frames are equivalent for the formulation of the general laws of physics. [1] This is called the equivalence principle.

Equivalence principle states that all reference-frames can be used as reference-frames with equal right and equal success in the formulation of the general laws of nature. General theory of relativity states that by application of arbitrary substitutions of the coordinate variables the physical equations must pass over into equations of the same form for each reference frame. Due to the equivalence principle of the general theory of relativity, every transformation between coordinate systems equally corresponds to the transition of one coordinate system into another.

Example for this can be that there is no difference in free fall in a gravitational field or being idle in a falling elevator, since in both cases there can be equally rightful taken the elevator as coordinate system or the earth and in the case of free falling, the falling object or the Earth.

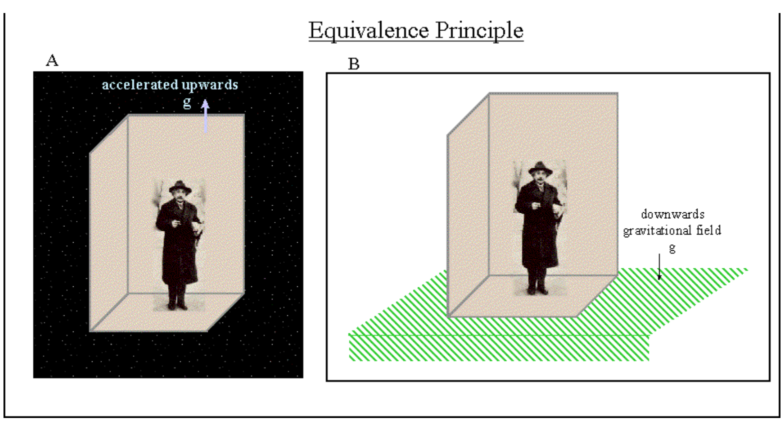

Figure 3. Equivalence principle between free fall and elevator use [3]

Energy and Space-time

Gravitation can be simplified as a curvature of spacetime, caused by presence of energy and matter. The curvature of spacetime is dependent of the mass and energy present: [8]

$$\boldsymbol{G}_{\mu v} + \Lambda \boldsymbol{g}_{\mu v} = \frac{8\pi G}{c^{\Lambda}} \boldsymbol{T}_{\mu v}$$

- $G_{\mu v}$ is the Einstein tensor which represents the curvature of spacetime in a certain point.
- Λ represents the cosmological constant.
- $g_{\mu v}$ is the metric tensor which represents the geometrical structure of spacetime.
- G is the Newton gravitational constant.
- $T_{\mu v}$ is the stress-energy tensor and it represents the flux and the density of energy and momentum in a certain point in space-time.
- c is the speed of light.

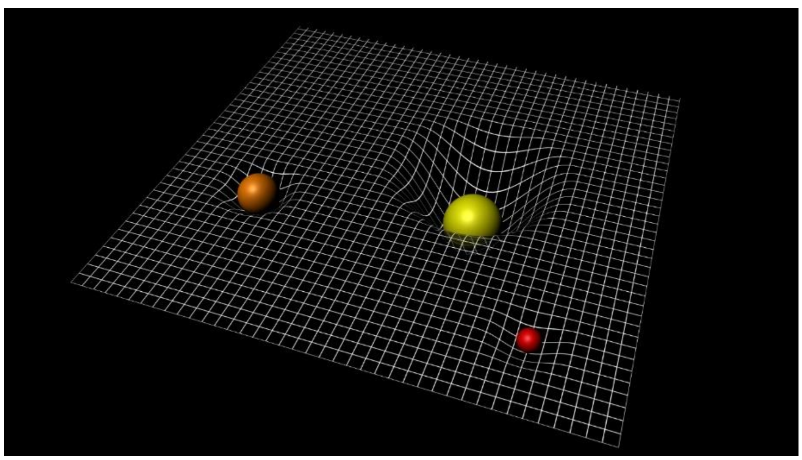

Figure 4. Space-time bending due to present energy [4]

The above picture illustrates intuitively the effect of energy on space-time. The gravitational field can be explained as curvature in the spacetime. This bending results in slower ticking of clocks due to gravitational field and the effect of light bending in which light rays are bent in the presence of gravitational field. The planetary movements around the sun are example for this effect of bending as rotation of planet around the sun is simply following the curvature of spacetime caused by Sun’s matter and energy.

Time travel possibility

Because of the bending of space-time due to the energy present, there exists a theoretical possibility for time travel if the space time is bent in such way that a closed loop to the future is formed.[9] This loop is called wormhole. In the below picture, this space-time bending would enable a ray of light to skip propagating through the bended space-time in order to get to the other side of the wormhole and just propagate through it, thus effectively being in the future. However, such time machine is nearly impossible since the amount of energy needed in order to bend time-space in such way is unforeseeable and is close to the magnitude of blackhole. This type of time-machine would be very unstable. No travelling in the spacetime before the bending was made will be possible. On a light note, this will help in avoiding eventual questions if history could be manipulated.

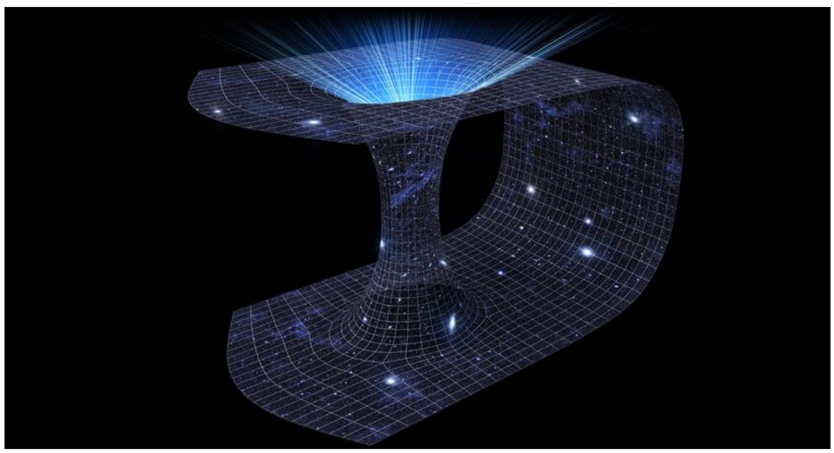

Figure 5. Wormhole as time travel possibility [5]

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[1] Albert Einstein. (1916). *Relativity: The Special and General Theory*. H. Holt and Company.

[2] Albert A. Michelson, Edward W. Morley. (1887). *On the relative motion of Earth and the luminous effect*. *American Journal of Science*.

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[8] Øyvind Grøn, Sigbjørn Hervik. (Version 9th, 2004 - Draft). *Einstein's General Theory of Relativity.* ©Grøn & Hervik.

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Image References:

[1] https://commons.wikimedia.org/wiki/File:Frames_of_reference_in_relative_motion.svg

[2] https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/5%3A__Relativity/5.3%3A_Time_Dilation

[3] http://astrosun2.astro.cornell.edu/academics/courses/astro201/equivalence_princ.htm

[4] https://www.esa.int/spaceinimages/Images/2015/09/Spacetime_curvature

[5] https://www.sciencenews.org/blog/context/new-einstein-equation-wormholes-quantum-gravity