A Top-down Approach to Combining Logics

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Outline/topics addressed

A Motivation:
combining logics, context, expressive ontologies

B Example:
first-order monomodal logic is a fragment of HOL
constant/varying/cumulative domain
first-order multimodal logic is a fragment of HOL
propositional quantification
bridge rules

C Many non-classical logics are natural fragments of HOL

D Proof automation

Combining logics

- prominent challenge in AI (CS, Philosophy)
- epistemic, deontic, temporal, intuitionistic, relevant, linear, conditional, security ...
- wide literature—few implementations
- some propositional systems exists: Logic Workbench, LoTREC, Tableaux Workbench, FaCT, ileanCoP, MSPASS
- no implemented systems for combinations of first-order logics
- combination is typically approached **bottom-up**

My approach is complementary:
works **top-down** starting from classical higher-order logic (HOL)

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Context

- prominent challenge in AI (CS, Philosophy)
- McCarthy: modeling of contexts as first-class objects

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ist(context_of("Ben's Knowledge),likes(Sue,Bill))

    ist(context_of("Ben's Knowledge),
        ist(context_of(...),...))
```

- McCarthy's approach has been followed by many others
- Giunchiglia emphasizes locality aspect; structured knowledge
- McCarthy and Giunchiglia **avoid modal logics**
- they also **avoid a HOL perspective**

My approach is complementary:
takes a **HOL perspective** and integrates **modal logics**

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Expressive ontologies

- SUMO and Cyc
- modeling of contexts:

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(holdsDuring (yearFn 2009 (loves Bill Mary)))

  (believes Bill
    (knows Ben
      (forall (?X)
        ((woman ?X) => (loves Bill ?X)))))
```

- relation to McCarthy's approach is obvious
- often a questionable semantics assumed for embedded formulas and modal predicates (also in Common Logic)

My approach:
HOL-based semantics, but holdsDuring, believes, knows and alike are associated with **modal logic connectives**

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A top-down approach to combining logics

- many non-classical logics are just natural fragments of HOL (via an elegant semantic embedding)
- they can be easily combined in HOL
- object-level reasoning enabled with off-the-shelf HOL provers and model finders
- even meta-level reasoning is feasible

Key idea of the approach:
Bridge between the Tarski view of logics (for meta-logic HOL) and the Kripke view of logics (for the embedded logics)

A: The Proposed Solution

A top-down approach to combining logics

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B: Example — Embedding of FML in HOL

First-order Modal Logics (FMLs)

$$p, q \quad ::= \quad P(t_1, \ldots, t_n) \mid (\neg p) \mid (p \vee q) \mid \Box p \mid (\forall x\, p)$$

are relevant for many applications, including

- planning
- natural language processing
- program verification
- modeling communication
- querying knowledge bases

Until recently, however, there has been

- a comparably large body of theory papers on FMLs
- but only one implemented prover! (GQML prover)

For recent progress see: [BenzmüllerOttenRaths, ECAI, 2012]

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Individuals

Booleans (True and False)

Functions/Predicates

B: Example — Embedding of FML in HOL

Simple Types $\alpha ::= \iota \mid \mu \mid o \mid \alpha_1 \rightarrow \alpha_2$

Possible worlds

Individuals

Booleans (True and False)

Functions/Predicates

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HOL $\quad s, t \;::=\; C_\alpha \mid x_\alpha \mid (\lambda x_\alpha\, s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid$
$(\neg_{o\to o}\, s_o)_o \mid (s_o \vee_{o\to o\to o} t_o)_o \mid (\forall x_\alpha\, t_o)_o$

Constant Symbols

Variable Symbols

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Variable Symbols
Abstraction
Application

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Logical Connectives

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HOL $\quad s, t \quad ::= \quad C \mid x \mid (\lambda x\, s) \mid (s\, t) \mid (\neg s) \mid (s \vee t) \mid (\forall x\, t)$

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HOL (with Henkin semantics) is meanwhile very well understood

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HOL TPTP Infrastructure

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HOL TPTP Infrastructure

HOL Provers: LEO-II, Satallax, TPS, Isabelle, Nitpick, agsyHOL

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FML $\quad p, q \quad ::= \quad P(t_1, \ldots, t_n) \mid (\neg p) \mid (p \vee q) \mid \Box p \mid (\forall x\, p)$

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\begin{array}{lll}
M, g, s \models \neg p & \text{iff} & \text{not } M, g, s \models p \\
M, g, s \models p \vee q & \text{iff} & M, g, s \models p \text{ or } M, g, s \models q \\
M, g, s \models \Box p & \text{iff} & M, g, u \models p \text{ for all } u \text{ with } R(s, u) \\
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FML in HOL:

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\begin{array}{lll}
\neg & = & \lambda p_{\iota \to o}\, \lambda w_\iota\, \neg p w \\
\vee & = & \lambda p_{\iota \to o}\, \lambda q_{\iota \to o}\, \lambda w_\iota\, (p w \vee q w) \\
\Box & = & \lambda p_{\iota \to o}\, \lambda w_\iota\, \forall v_\iota\, (\neg R w v \vee p v) \\
\Pi & = & \lambda h_{\mu \to (\iota \to o)}\, \lambda w_\iota\, \forall x_\mu\, h x w
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$$

now $\forall x p$ stands for $\Pi \lambda x p$

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Meta-level notions: valid $= \lambda p_{\iota\to o}\, \forall w_\iota\, pw$

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Main idea: Lifting of modal formulas to predicates on worlds

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$$(\Diamond \exists x P f x \wedge \Box \forall y (\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond \exists z Q z$$

$$\text{valid } (\Diamond \exists x P f x \wedge \Box \forall y (\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond \exists z Q z$$

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...

$$\forall w(\neg\neg(\neg\neg\forall v(\neg Rwv \vee \neg\neg\forall x \neg P(fx)v) \vee \neg\forall v(\neg Rwv \vee \forall y(\neg\neg\forall u(\neg Rvu \vee \neg Pyu) \vee Qyv))) \vee \neg\forall v(\neg Rwv \vee \neg\neg\forall z \neg Qzv))$$

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Axiomatization of properties of accessibility relation R

Logic K:	no axioms
Logic T:	$(\mathit{reflexive}\ R)$ — which expands into $\forall x\ R x x$
Logic S4:	$(\mathit{reflexive}\ R) \wedge (\mathit{symmetric}\ R) \wedge (\mathit{transitive}\ R)$
Logic ...	...

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Logic ...	...

This automates FML with <u>constant domain semantics</u> in HOL

B: Example — Embedding FML in HOL

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To obtain varying domain semantics:

- modify quantifier: $\Pi = \lambda q \lambda w \forall x \, \mathsf{ExistsInW} x w \Rightarrow q x w$
- add non-emptiness axiom: $\forall w \, \exists x \mathsf{ExistsInW} x w$
- add designation axioms for constants c: $\forall w \, \mathsf{ExistsInW} c w$
  (similar for function symbols)

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  (similar for function symbols)

To obtain cumulative domain semantics:

- add axiom: $\forall x \forall v \forall w\, \mathsf{ExistsInW} x v \wedge R v w \Rightarrow \mathsf{ExistsInW} x w$

What extras are needed?

- instead of $\Box = \lambda p\, \lambda w\, \forall v(\neg(Rwv) \vee (pv))$
  consider $\Box = \lambda r\, \lambda p\, \lambda w\, \forall v(\neg(rwv) \vee (pv))$
- now we may have: $\Box_{knowledgeBen}, \Box_{commonKnowledge}, \cdots$
- we can add quantification over propositional variables

$$\Pi^p = \lambda q_{(\iota\to o)\to(\iota\to o)}\, \lambda w_\iota\, \forall p_{\iota\to o}(qpw) \quad (\forall pq \text{ stands for } \Pi^p \lambda pq)$$

- and use this to explicitly encode bridge rules

$$\forall p\, (\Box_{commonKnowledge}\, p \supset \Box_{knowledgeBen}\, p)$$

What can we do with that?

- actually a lot
- see e.g. the elegant modeling and effective solution of the Wise Men Puzzle as reported in the paper

C: Many Non-classical Logics are Fragments of HOL

Soundness and completeness

$$\models \varphi \quad \text{iff} \quad \models^{HOL} \text{valid}\, \varphi_{\iota\to o}$$

results do already exist for

- propositional multimodal logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- quantified multimodal logics [BenzmüllerPaulson, Logica Universalis, 2012]
- propositional conditional logics [BenzmüllerEtAl., AMAI, 2012]
- intuitionistic logics: [BenzmüllerPaulson, Log.J.IGPL, 2010]
- access control logics: [Benzmüller, IFIP SEC, 2009]
- combinations of logics: [Benzmüller, AMAI, 2011]

- . . . more is on the way . . .

C: Why Not Throwing Things Together?

Terms: $m ::= C \mid x \mid (F\, m^1 \ldots m^n)$

Formulas: $s, t ::= (k\, m^1 \ldots m^n) \mid \neg s \mid s \vee t \mid \Box_r s \mid s \Rightarrow_f t \mid \ldots$
$\forall x\, s \mid \forall_{vary} x\, s \mid \forall_{cumul} x\, s \mid \forall^p p\, s \mid \ldots$

Embedding in HOL:

$$C = C_\mu \qquad x = x_\mu \qquad F = F_{\mu^n \to \mu}$$

$$k = k_{\mu^n \to \iota \to o}$$

$$r = r_{\iota \to \iota \to o} \quad (+axioms\ for\ r) \qquad f = f_{\iota \to \iota \to o} \quad (+axioms\ for\ f)$$

$$\neg = \lambda s_{\iota \to o}\, \lambda w_\iota\, \neg sw)$$

$$\vee = \lambda s_{\iota \to o}\, \lambda t_{\iota \to o}\, \lambda w_\iota\, (sw \vee tw)$$

$$\Box = \lambda r_{\iota \to \iota \to o}\, \lambda s_{\iota \to o}\, \lambda w_\iota\, \forall v_\iota\, \neg rwv \vee sv$$

$$\Rightarrow = \lambda f_{\iota \to (\iota \to o)\iota \to o}\, \lambda s_{\iota \to o}\, \lambda t_{\iota \to o}\, \lambda w_\iota\, \forall v_\iota\, (\neg fwsv \vee tv)$$

$$\Pi = \lambda q_{\mu \to (\iota \to o)}\, \lambda w_\iota\, \forall x_\mu\, qxw$$

$$\Pi_{var/cumul} = \lambda q_{\mu \to (\iota \to o)}\, \lambda w_\iota\, \forall x_\mu\, \neg \mathrm{exInW} xw \vee qxw$$

$$\Pi^p = \lambda q_{(\iota \to o) \to (\iota \to o)}\, \lambda w_\iota\, \forall p_{\iota \to o}\, qpw$$

$\ldots$ further non-classical connect., quantif. over higher types, predicate abstraction, definite description ...

D: Proof Automation — How Competitive is HOL?

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FML Experiment: **580 problems** × **5 logics** × **3 domain cond.** × **6 provers** × **600s tmo**

8700 problems

Logic/ Domain	f2p-MSPASS v3.0	MleanSeP v1.2	LEO-II v1.4.2	Satallax v2.2	MleanTAP v1.3	MleanCoP v1.2
	ATP system					
K/varying	-	-	72	104	-	-
K/cumul.	70	121	89	122	-	-
K/constant	67	124	120	146	-	-
D/varying	-	-	128 81	113	100	179
D/cumul.	79	130	144 100	133	120	200
D/constant	76	134	167 135	160	135	217
T/varying	-	-	170 120	170	138	224
T/cumul.	105	163	190 139	192	160	249
T/constant	95	166	217 173	213	175	269
S4/varying	-	-	140	207	169	274
S4/cumul.	121	197	218 166	238	205	338
S4/constant	111	197	244 200	261	220	352
S5/varying	-	-	169	248	219	359
S5/cumul.	140	-	215	297	272	438
S5/constant	131	-	237	305	272	438

D: Proof Automation — How Competitive is HOL?

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FML Experiment: **580 problems** × **5 logics** × **3 domain cond.** × **6 provers** × **600s tmo**

Logic/ Domain	f2p-MSPASS v3.0	MleanSeP v1.2	LEO-II v1.3.2	Satallax v2.2	MleanTAP v1.3	MleanCoP v1.2
K/varying	-	-	72	104	-	-
K/cumul.	70	121	89	122	-	-
K/constant	67	124	120	146	-	-
D/varying	-	-	81	113	100	179
D/cumul.	79	130	100	133	120	200
D/constant	76	134	135	160	135	217
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Strongest Prover!
A specialist system.

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Logic/ Domain	ATP system					
	f2p-MSPASS v3.0	MleanSeP v1.2	LEO-II v1.4.2	Satallax v2.2	MleanTAP v1.3	MleanCoP v1.2
K/varying	-	-	72	104	-	-
K/cumul.	70	121	89	122	-	-
K/constant	67	124	120	146	-	-
D/varying	-	-	128 81	113	100	179
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HOL provers, 2nd best
Strong recent improvements

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Results for 20 multimodal logic problems: LEO-II 15, Satallax 14

Summary

I have argued that:

- many non-classical logics are natural fragments of HOL
- they can easily be combined in HOL
- they can be automated in HOL (object-level and meta-level)
- automation of HOL is currently making good progress
- **we get reasoners for expressive non-classical logics (and their combinations) for free**
- for many of those no practical systems are available yet
- this is relevant for: context and expressive ontologies

Ongoing & future work:

- automation of expressive ontologies, e.g. SUMO
- proper semantics for SUMO
- further applications

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