A Top-down Approach to Combining Logics

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Outline/topics addressed

A Motivation:
combining logics, context, expressive ontologies

B Example:
first-order monomodal logic is a fragment of HOL
constant/varying/cumulative domain
first-order multimodal logic is a fragment of HOL
propositional quantification
bridge rules

C Many non-classical logics are natural fragments of HOL

D Proof automation
A: Motivation

Combining logics

- prominent challenge in AI (CS, Philosophy)
- epistemic, deontic, temporal, intuitionistic, relevant, linear, conditional, security . . .
- wide literature—few implementations
- some propositional systems exist: Logic Workbench, LoTREC, Tableaux Workbench, FaCT, ileanCoP, MSPASS
- no implemented systems for combinations of first-order logics
- combination is typically approached bottom-up

My approach is complementary:
works top-down starting from classical higher-order logic (HOL)
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Context

- prominent challenge in AI (CS, Philosophy)
- McCarthy: modeling of contexts as first-class objects

\[
\text{ist(context\_of("Ben's Knowledge"),likes(Sue,Bill))}
\]

\[
\text{ist(context\_of("Ben's Knowledge"),}
\text{ist(context\_of(...),...))}
\]

- McCarthy’s approach has been followed by many others
- Giunchiglia emphasizes locality aspect; structured knowledge
- McCarthy and Giunchiglia avoid modal logics
- they also avoid a HOL perspective

My approach is complementary:
takes a HOL perspective and integrates modal logics
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- they also \textit{avoid a HOL perspective}

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A: Motivation

Expressive ontologies

- SUMO and Cyc
- modeling of contexts:

\[
\text{(holdsDuring (yearFn 2009 (loves Bill Mary)))}
\]

\[
\text{(believes Bill}
\text{)
\text{(knows Ben}
\text{(forall (?X)
\text{ ((woman ?X) => (loves Bill ?X)))))}
\]

- relation to McCarthy’s approach is obvious
- often a questionable semantics assumed for embedded formulas and modal predicates (also in Common Logic)

My approach:
HOL-based semantics, but holdsDuring, believes, knows and alike are associated with modal logic connectives
A: Motivation

Expressive ontologies

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A: The Proposed Solution

A top-down approach to combining logics

- many non-classical logics are just natural fragments of HOL (via an elegant semantic embedding)
- they can be easily combined in HOL
- object-level reasoning enabled with off-the-shelf HOL provers and model finders
- even meta-level reasoning is feasible

Key idea of the approach:
Bridge between the Tarski view of logics (for meta-logic HOL) and the Kripke view of logics (for the embedded logics)
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Bridge between the Tarski view of logics (for meta-logic HOL) and the Kripke view of logics (for the embedded logics)
First-order Modal Logics (FMLs)

\[ p, q ::= P(t_1, \ldots, t_n) \mid (\neg p) \mid (p \lor q) \mid \Box p \mid (\forall x p) \]

are relevant for many applications, including

- planning
- natural language processing
- program verification
- modeling communication
- querying knowledge bases

Until recently, however, there has been

- a comparably large body of theory papers on FMLs
- but only one implemented prover! (GQML prover)

For recent progress see: [BenzmüllerOttenRaths, ECAI, 2012]
B: Example — Embedding of FML in HOL

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Simple Types

\[ \alpha ::= \iota \mid \sigma \mid \alpha_1 \rightarrow \alpha_2 \]
B: Example — Embedding of FML in HOL

Simple Types

Individuals

Boolean (True and False)

Functions/Predicates

\[ \alpha ::= \iota \mid o \mid \alpha_1 \rightarrow \alpha_2 \]
B: Example — Embedding of FML in HOL

Simple Types

\[ \alpha ::= \iota \mid \mu \mid o \mid \alpha_1 \rightarrow \alpha_2 \]

Possible worlds

Individuals

Booleans (True and False)

Functions/Predicates
B: Example — Embedding of FML in HOL

\[ s, t ::= C_{\alpha} | x_{\alpha} | (\lambda x_{\alpha} s_{\beta})_{\alpha \rightarrow \beta} | (s_{\alpha \rightarrow \beta} t_{\alpha})_{\beta} | (\neg o \rightarrow o s_{o})_{o} | (s_{o \rightarrow o o} t_{o})_{o} | (\forall x_{\alpha} t_{o})_{o} \]
B: Example — Embedding of FML in HOL

\[ s, t ::= C_\alpha | x_\alpha | (\lambda x_\alpha s_\beta)_{\alpha \rightarrow \beta} | (s_{\alpha \rightarrow \beta} t_\alpha)_\beta | (\neg o_{\rightarrow o} s_0)_o | (s_0 \lor o_{\rightarrow o} o t_0)_o | (\forall x_\alpha t_0)_o \]
B: Example — Embedding of FML in HOL

\[
\begin{align*}
\text{HOL} & : s, t ::= C_\alpha | x_\alpha | (\lambda x_\alpha s_\beta)_\alpha \to \beta | (s_\alpha \to t_\alpha)_\beta \\
& | (\neg o \to o s_o)_o | (s_o \lor o \to o t_o)_o | (\forall x_\alpha t_o)_o
\end{align*}
\]

Constant Symbols
Variable Symbols
Abstraction
Application
Logical Connectives
Example — Embedding of FML in HOL

\[ \text{HOL} \quad s, t \ ::= \quad C_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha \rightarrow \beta} \mid (s_{\alpha \rightarrow \beta} t_\alpha)_{\beta} \mid \]
\[ \quad (\neg_{o \rightarrow o} s_o)_{o} \mid (s_o \lor_{o \rightarrow o \rightarrow o} t_o)_{o} \mid (\forall x_\alpha t_o)_{o} \]
B: Example — Embedding of FML in HOL

\[
\text{HOL} \quad s, t := C \mid x \mid (\lambda x \ s) \mid (s \ t) \mid (\neg s) \mid (s \lor t) \mid (\forall x \ t)
\]
B: Example — Embedding of FML in HOL

HOL   \[ s, t ::= C \mid x \mid (\lambda x \ s) \mid (s \ t) \mid (\neg s) \mid (s \lor t) \mid (\forall x \ t) \]

HOL (with Henkin semantics) is meanwhile very well understood
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\text{HOL} & \quad s, t ::= C \mid x \mid (\lambda x \; s) \mid (s \; t) \mid (\neg s) \mid (s \; \lor \; t) \mid (\forall x \; t) \\
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HOL TPTP Infrastructure
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HOL TPTP Infrastructure

HOL Provers: LEO-II, Satallax, TPS, Isabelle, Nitpick, agsyHOL
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\text{FML} & \quad p, q \ ::= \ P(t_1, \ldots, t_n) \mid (\neg p) \mid (p \lor q) \mid (p) \mid (\forall x \ p)
\end{align*}
\]

\[
\begin{align*}
M, g, s \models \neg p & \quad \text{iff} \quad \text{not } M, g, s \models p \\
M, g, s \models p \lor q & \quad \text{iff} \quad M, g, s \models p \text{ or } M, g, s \models q \\
M, g, s \models (p) & \quad \text{iff} \quad M, g, u \models p \text{ for all } u \text{ with } R(s, u) \\
M, g, s \models (\forall x \ p) & \quad \text{iff} \quad M, [d/x]g, s \models p \text{ for all } d \in D
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\[ p, q \ ::= \ P(t_1, \ldots, t_n) \mid (\neg p) \mid (p \lor q) \mid \Box p \mid (\forall p) \]

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\[ M, g, s \models \forall x p \quad \text{iff} \quad M, [d/x]g, s \models p \text{ for all } d \in D \]

FML in HOL:
\[ \neg = \lambda p \o \lambda w \ o \lambda w (\neg pw) \]
\[ \lor = \lambda p \o \lambda q \o \lambda w (pw \lor qw) \]
\[ \Box = \lambda p \o \lambda w \ \forall v (\neg Rwv \lor pv) \]
\[ \Pi = \lambda h \mu \to (\o \to) \lambda w \ \forall x \mu hxw \]

now \( \forall x p \) stands for \( \Pi \lambda x p \)
B: Example — Embedding of FML in HOL

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\text{FML in HOL:} \quad \neg = \lambda p_{l \to o} \lambda w_l \neg pw
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\text{Meta-level notions:} \quad \text{valid} = \lambda p_{l \to o} \forall w_l pw
\]
B: Example — Embedding of FML in HOL

HOL $s, t ::= C \mid x \mid (\lambda x \ s) \mid (s \ t) \mid (\neg s) \mid (s \lor t) \mid (\forall x \ t)$

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Meta-level notions: $\text{valid} = \lambda p_{\rightarrow o} \forall w_{l} pw$

Main idea: Lifting of modal formulas to predicates on worlds
B: Example — Embedding of FML in HOL

\[(\Diamond \exists x Pf x \land \Box \forall y (\Diamond Py \Rightarrow Qy)) \Rightarrow \Diamond \exists z Qz\]

valid \[(\Diamond \exists x Pf x \land \Box \forall y (\Diamond Py \Rightarrow Qy)) \Rightarrow \Diamond \exists z Qz\]
B: Example — Embedding of FML in HOL

(◇∃xPx ∧ □∀y(◇Py ⇒ Qy)) ⇒ ◇∃zQz

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\[\Box = \lambda p \lambda w \forall v (\neg (Rwv) \lor (pv))\]
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valid \((\Diamond \exists x Pfx \land (\lambda w \forall v (~ (Rwv) \lor (\forall y (\Diamond Py \Rightarrow Qy) v)))) \Rightarrow \Diamond \exists z Qz\)

\(\Box = \lambda p \lambda w \forall v (~ (Rwv) \lor (pv))\)
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valid (◊∃xPx ∧ (λw ∀v(¬(Rvw) ∨ (∀y(◊Py⇒Qy)v)))) ⇒ ◊∃zQz

∀w(¬¬(¬¬∀v(¬Rvw ∨ ¬¬∀x¬P(fx)v) ∨ ¬∀v(¬Rvw ∨ ∀y(¬¬∀u(¬Rvu ∨ ¬Pyu) ∨ Qyv)))) ∨ ¬∀v(¬Rvw ∨ ¬∀z¬Qzv))
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\[ \forall w (\neg \neg \forall v (\neg Rwv \lor \neg \forall x \neg P(fx)v) \lor \neg \forall v (\neg Rwv \lor \forall y (\neg \neg \forall u (\neg Rvu \lor \neg Pyu) \lor Qyv)))) \lor \neg \forall v (\neg Rwv \lor \neg \forall z \neg Qzv)) \]
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Axiomatization of properties of accessibility relation \(R\)

Logic K: no axioms
Logic T: (reflexive \(R\)) — which expands into \(\forall x Rxx\)
Logic S4: (reflexive \(R\)) \land (symmetric \(R\)) \land (transitive \(R\))
Logic . . . . .
B: Example — Embedding of FML in HOL

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\[
\forall w (\neg \neg (\neg \neg \forall v (\neg Rwv \lor \neg \forall x \neg P(fx)v) \lor \neg \forall v (\neg Rwv \lor \forall y (\neg \neg \forall u (\neg Rvu \lor \neg Pyu) \lor Qyv)))) \lor \neg \forall v (\neg Rwv \lor \neg \forall z \neg Qzv))
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Logic . . . . . .

This automates FML with constant domain semantics in HOL
To obtain varying domain semantics:

- modify quantifier: \[ \Pi = \lambda q \lambda w \forall x \text{ExistsInW}xw \Rightarrow qxw \]
- add non-emptiness axiom: \[ \forall w \exists x \text{ExistsInW}xw \]
- add designation axioms for constants \( c \): \[ \forall w \text{ExistsInW}cw \]
  (similar for function symbols)
To obtain varying domain semantics:

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- add designation axioms for constants \( c \):
  \[ \forall w \text{ExistsInW}cw \]
  (similar for function symbols)

To obtain cumulative domain semantics:

- add axiom:
  \[ \forall x \forall v \forall w \text{ExistsInW}xv \land Rvw \Rightarrow \text{ExistsInW}xw \]
B: Example — First-order Multimodal Logics in HOL

What extras are needed?

- instead of $\square = \lambda p \lambda w \forall v (\neg (Rwv) \lor (pv))$
  consider $\square = \lambda r \lambda p \lambda w \forall v (\neg (rwv) \lor (pv))$

- now we may have: $\square_{knowledgeBen}, \square_{commonKnowledge}, \cdots$

- we can add quantification over propositional variables

$$\Pi^p = \lambda q_{(l \rightarrow o) ightarrow (l \rightarrow o)} \lambda w_l \forall p_{l \rightarrow o} (qpw) \quad (\forall pq \text{ stands for } \Pi^p \lambda pq)$$

- and use this to explicitly encode bridge rules

$$\forall p (\square_{commonKnowledge} p \supset \square_{knowledgeBen} p)$$

What can we do with that?

- actually a lot

- see e.g. the elegant modeling and effective solution of the Wise Men Puzzle as reported in the paper
C: Many Non-classical Logics are Fragments of HOL

Soundness and completeness

\[ \models \varphi \text{ iff } \models^{HOL} \text{valid } \varphi_{\rightarrow_0} \]

results do already exist for

- quantified multimodal logics [BenzmullerPaulson, Logica Universalis, 2012]
- propositional conditional logics [BenzmullerEtAl., AMAI, 2012]
- access control logics: [Benzmuller, IFIP SEC, 2009]
- combinations of logics: [Benzmuller, AMAI, 2011]

...more is on the way...
C: Why Not Throwing Things Together?

Terms:

\[ m ::= C \mid x \mid (F \ m^1 \ldots m^n) \]

\[ s, t ::= (k \ m^1 \ldots m^n) \mid \neg s \mid s \lor t \mid \Box_r s \mid s \Rightarrow_f t \mid \ldots \]

Formulas:

\[ \forall x \ s \mid \forall_{\text{vary}} x \ s \mid \forall_{\text{cumul}} x \ s \mid \forall^p p \ s \mid \ldots \]

Embedding in HOL:

\[
\begin{align*}
C &= C_{\mu} \\
x &= x_{\mu} \\
F &= F_{\mu^n \rightarrow \mu} \\
k &= k_{\mu^n \rightarrow \iota \rightarrow o} \\
r &= r_{\iota \rightarrow \iota \rightarrow o} \quad (+\text{axioms for } r) \\
f &= f_{\iota \rightarrow \iota \rightarrow o} \quad (+\text{axioms for } f) \\
\neg &= \lambda s_{\iota \rightarrow o} \lambda w_{\iota} \neg sw \\
\lor &= \lambda s_{\iota \rightarrow o} \lambda t_{\iota \rightarrow o} \lambda w_{\iota} (sw \lor tw) \\
\Box &= \lambda r_{\iota \rightarrow \iota \rightarrow o} \lambda s_{\iota \rightarrow o} \lambda w_{\iota} \forall v_{\iota} \neg r w v \lor sv \\
\Rightarrow &= \lambda f_{\iota \rightarrow (\iota \rightarrow o) \iota \rightarrow o} \lambda s_{\iota \rightarrow o} \lambda t_{\iota \rightarrow o} \lambda w_{\iota} \forall v_{\iota} (\neg f w s v \lor tv) \\
\Pi &= \lambda q_{\mu \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall x_{\mu} q x w \\
\Pi_{\text{var}} &= \lambda q_{\mu \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall x_{\mu} \neg \text{exInW} x w \lor q x w \\
\Pi^p &= \lambda q_{(\iota \rightarrow o) \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall p_{\iota \rightarrow o} q p w \\
\ldots & \text{ further non-classical connect., quantif. over higher types,} \\
& \text{predicate abstraction, definite description} \ldots
\end{align*}
\]
**D: Proof Automation — How Competitive is HOL?**

FML Experiment: 580 problems $\times$ 5 logics $\times$ 3 domain cond. $\times$ 6 provers $\times$ 600s tmo

8700 problems

<table>
<thead>
<tr>
<th>Logic/Domain</th>
<th>f2p-MSPASS v3.0</th>
<th>MleanSeP v1.2</th>
<th>LEO-II v1.4.2</th>
<th>Satallax v2.2</th>
<th>MleanTAP v1.3</th>
<th>MleanCoP v1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>K/varying</td>
<td>-</td>
<td>-</td>
<td>72</td>
<td>104</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>K/cumul.</td>
<td>70</td>
<td>121</td>
<td>89</td>
<td>122</td>
<td>-</td>
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### D: Proof Automation — How Competitive is HOL?

FML Experiment: **580 problems × 5 logics × 3 domain cond. × 6 provers × 600s tmo**

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**Strongest Prover!**

A specialist system.

C. Benzmüller, 2013 —— A Top-down Approach to Combining Logics —— ICAART
### D: Proof Automation — How Competitive is HOL?

**FML Experiment:** 580 problems \( \times \) 5 logics \( \times \) 3 domain cond. \( \times \) 6 provers \( \times \) 600s tmo

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**HOL provers, 2nd best**

**Strong recent improvements**
D: Proof Automation — How Competitive is HOL?

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Results for 20 multimodal logic problems: LEO-II 15, Satallax 14
Summary

I have argued that:

▸ many non-classical logics are natural fragments of HOL
▸ they can easily be combined in HOL
▸ they can be automated in HOL (object-level and meta-level)
▸ automation of HOL is currently making good progress
▸ **we get reasoners for expressive non-classical logics (and their combinations) for free**
▸ for many of those no practical systems are available yet
▸ this is relevant for: context and expressive ontologies

Ongoing & future work:

▸ automation of expressive ontologies, e.g. SUMO
▸ proper semantics for SUMO
▸ further applications
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