The CALCULEMUS Research Training Network
(HPRN-CT-2000-00102)

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CALCULEMUS

Interest Group
since mid 90s
www.calkulemus.org

EU Research Training Network
09/2000 – 09/2004
www.eurice.de/calkulemus/
Scientific Motivation

New generation of (mathematical) assistant systems
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New generation of (mathematical) assistant systems

- Integration of symbolic reasoning and symbolic computation

- Applications in mathematics, maths education, formal methods
Scientific Motivation

New generation of (mathematical) assistant systems

- Integration of symbolic reasoning and symbolic computation
- Interoperability with mathematical knowledge bases
- Integration of heterogeneous specialist reasoners
- Open system architectures and mathematical services
- Applications in mathematics, maths education, formal methods

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Scientific Motivation

New generation of (mathematical) assistant systems

- Integration of symbolic reasoning and symbolic computation
- Interoperability with mathematical knowledge bases
- Knowledge exploration, maintenance, management of change
- Integration of heterogeneous specialist reasoners
- Expressive representations; human-oriented user interfaces
- Support for representation transformations
- Open system architectures and mathematical services
- Preparation and validation of mathematical texts and publications
- Applications in mathematics, maths education, formal methods
Sociological Goal

Early stage training of young researchers
Sociological Goal

Early stage training of young researchers

Measures:

- The CALCULEMUS Autumn School 2002
- CALCULEMUS Symposia and Network Meetings
- Training at an Individual Level at the Network Nodes
- Local Courses, Workshops, Talks, and Seminars
- Exchange of YVRs between Network Nodes
- Industry Internships
The CALCULEMUS RTN

USAAR  Saarbrücken  Siekmann
UED  Edinburgh  Bundy
UKA  Karlsruhe  Calmet
RISC  Linz  Buchberger
TUE  Eindhoven/Nijmegen  Cohen/Barendregt
ITC-IRST  Trento  Giunchiglia
UWB  Bialystok  Trybulec
UGE  Genova  Armando
UBIR  Birmingham  Kerber

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CALCULEMUS Methodology

Vision: Powerful Mathematical Assistant System(s)

Reality: heterogeneous frameworks, systems, and tools
individual strengths and weaknesses

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**CALCULEMUS Methodology**

**Vision:** Powerful Mathematical Assistant System(s)

**Bottom-up from:** CAS  DS  Math-KBs  ...  

**Possible cooperation with:** QPQ?
# CAS & DS: The Map

<table>
<thead>
<tr>
<th>DS ⊆ CAS:</th>
<th>CAS ⊆ DS:</th>
</tr>
</thead>
</table>
| - **THEOREMA** ⊆ *Mathematica* | - (tight coupling:  
   - T-unification, constraint resolution, T-resolution) |
| - HR uses **Otter** for **Maple** | - loose coupling:  
   - reflection approach as used in **Coq**  
   - proof planning (**\Clam**, **MEGA***) |
| CAS ≡ DS: | protocol, e.g. á la Calmet |
|           | common interface:  
   - top down: OMRS, MathWeb-SB, LBA, MathBroker  
   - bottom up: CCR, MathSat |

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Experiences

Bad news:

- no single predominant approach for CAS & DS
Experiences

Bad news:

- no single predominant approach for CAS & DS

Good news:

- heterogeneity is not necessarily bad
- challenge is to support heterogeneity
- frameworks supporting the integration of heterogeneous tools are in development (CAS ≡ DS)
CAS and ATP in Proof Planning

Proof Planning (as an example for \( \text{CAS} \subseteq \text{DS} \)):

domain specific, heuristic reasoning at abstract layer

Integration of Specialist Reasoners (CASs and ATPs):

- at method layer
- at the heuristic meta-reasoning layer
CAS and ATP in Proof Planning

soundness is evaluated by …
CAS and ATP in Proof Planning

Proof Planning (as an example for CAS):
- domain specific, heuristic reasoning at abstract layer

refinement (expansion) over several layers

...
CAS and ATP in Proof Planning

probably supported by external reasoners . . .
CAS and ATP in Proof Planning

Proof Planning (as an example for CAS):
- domain specific, heuristic reasoning at abstract layer
- final verification at elementary logic layer

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CAS and ATP in Proof Planning

Required/Useful for $\text{CAS} \subseteq \text{DS}$:

- white box integration of external specialist reasoners
- tools for extraction and transformation of results

Mathematical Assistant System

Transformation Tool

<table>
<thead>
<tr>
<th>Problem</th>
<th>Proof Transformation</th>
<th>Proof Reconstruction</th>
<th>Problem'</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e.g. ND)</td>
<td>(Sapper, Tramp)</td>
<td></td>
<td>result (cryptic)</td>
</tr>
</tbody>
</table>

External Specialist Reasoner

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QPQ and CALCULEMUS?

Short-term

- central repository for tools
- foster uniform (problem and proof) representations
- provide (problem and proof) transformation tools
QPQ and CALCULEMUS?

**Short-term**
- central repository for tools
- foster uniform (problem and proof) representations
- provide (problem and proof) transformation tools

**Long-term**
- foster semantical descriptions of tools
- cooperate with emerging semantic brokering mechanism
## An ATP Service in MSDL

Source: Jürgen Zimmer (Edingurgh/Saarbrücken)

<table>
<thead>
<tr>
<th>Service: SpassProver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>classification:</strong></td>
</tr>
<tr>
<td>Classifi cation with Taxonomy of services or link to Ontology (→ QPQ)</td>
</tr>
<tr>
<td>→ first-order problem description</td>
</tr>
<tr>
<td><strong>service interface:</strong></td>
</tr>
<tr>
<td>→ fo-prover.wsd1</td>
</tr>
<tr>
<td><strong>implementation details:</strong></td>
</tr>
<tr>
<td>Information about hardware, software (calculus, etc.)</td>
</tr>
</tbody>
</table>

### first-order problem description

| input parameters: | name: *problem*, signature: ATP-Problem (DAML-S Class) |
| output parameters: | name: *result*, signature: ATP-Result (DAML-S Class) |
| pre-conditions: | `equational_reasoning(problem)` |
|                  | `∧ Ax = axioms(problem)` |
|                  | `∧ C = conjecture(problem)` |
|                  | `∧ ∀a ∈ Ax.first_order(a)` |
|                  | `∧ first_order(C)` |
|                  | `...` |
| post-conditions: | `Ax ⊢ \text{FOL } C : has(result, proof_object)` |

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Given: HO problem
Want: ND proof object

\[ \text{TRANS} \]

\[ \text{HO-ATP} \]

\[ \text{FO-ATP}_1 \]

\[ \text{FO-ATP}_2 \]

\[ \text{ND proof} \]

\[ \text{ND proof} \]
Given: HO problem
Want: Natural Language Proof

TRANS

FO-ATP₁
FO-ATP₂

HO-ATP

ND proof

NL-Gener.

ND proof

NL proof

NL proof

NL proof

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Related (EU) research initiatives

- **MONET: Mathematics on the Net**
  *offering mathematical algorithms through web services*

- **MOWGLI: Mathematics on the Web: Get it by Logics and Interfaces**
  *from machine-readable to machine-understandable representations of mathematical information*

- **OpenMath:**
  *standard for representing mathematical objects with their semantics*

- **MKM: Mathematical Knowledge Management Network**
  *from paper-oriented and presentation-oriented view to a semantics-oriented view of mathematical knowledge*

...