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Directional Reflectance and Emissivity of an Opaque Surface

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DIRECTIONAL REFLECTANCE AND EMISSIVITY OF AN OPAQUE SURFACE

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DIRECTIONAL REFLECTANCE AND EMISSIVITY OF AN OPAQUE SURFACE

Fred E. Nicodemus

1. ABSTRACT.

Concepts, terminology, and symbols are presented for specifying and relating directional variations in reflectance and emissivity of an opaque surface element. Their relationship to more familiar concepts, including those of perfectly diffuse and specular reflectance, is given, and they are applied to illustrative examples. It is shown that, when the usual reciprocity relationship holds, the reflectance for a ray incident on an opaque surface element is related by Kirchhoff's Law to the emissivity of that element for a ray emitted along the same line in the opposite sense.

2. INTRODUCTION.

Reflectance and emissivity of the surface of an opaque body are considered as properties of the surface material and of its microscopic configuration (roughness) but not of its gross configuration (curvature). This distinction between microscopic and gross details of the surface configuration, which is to some extent an arbitrary one, will be discussed further below. But reflectance and emissivity are commonly defined or specified in ways which include an implicit (and often overlooked) dependence on the geometry of the radiation beam (including incident, emitted, and reflected rays and the effects on those rays of the gross surface features). Even when this dependence is recognized, the specified reflectance or emissivity is usually applicable only to situations which reproduce the same geometry. On the other hand, it is possible to specify the reflectance and emissivity of an opaque surface (i.e., of any planar surface element) concisely and unambiguously as functions of direction (with reference to the orientation of the surface element) which can be applied quite generally.

The purpose of this paper is, first, to describe such a way of specifying the directional reflectance and emissivity of an opaque surface, to recommend appropriate terminology and symbols, and to relate them to those in common use. Second, a relationship will be established between the directional reflectance of a surface element (i.e., its reflectance for a ray incident from a particular direction) and the directional emissivity of the surface element for radiation emitted in that same direction. In other words, the related quantities are the reflectance for a ray incident along a line which intersects the surface element and the emissivity for a ray emitted along the same line in the opposite sense.

The radiometric quantities used are listed in Table I, reproduced from an earlier paper[1]. The radiometric relations will be analyzed below primarily in terms of the basic quantity radiance (N). In the earlier paper[1] it was shown that when radiance is defined, as in Table I, as the radiant flux or power (P) per unit solid-angle (Ω) - in-the-direction-of-a-ray per unit projected-area ($A\cos\theta$)-perpendicular-to-the-ray, it has the same value at any point along this ray within an isotropic medium, in the absence of losses by absorption, scattering, or reflection. More generally, the quantity N/n^2 (where n is the index of refraction) in the direction of a ray was shown to be invariant along that ray, even across a smooth boundary between different lossless media.

1. See list of references in Section 8.

TABLE I. Radiometric quantities, symbols, definitions, and units.*

Quantity	Symbol	Defining relations	Units
Radiant energy	U		J
Radiant energy density	u	$u = \frac{\partial U}{\partial V}$	$\mathrm{J \cdot cm^{-3}}$
Radiant power	P	$P = \frac{\partial U}{\partial t}$	watt (W)
Radiant intensity	J	$J = \frac{\partial P}{\partial \Omega}$	$\mathrm{W \cdot sr^{-1}}$
Radiant emittance	W	$\left.\begin{matrix} W \\ H \end{matrix}\right\} = \frac{\partial P}{\partial A}$	$\mathrm{W \cdot cm^{-2}}$
Irradiance	H		
Radiance	N	$N = \frac{\partial^2 P}{\cos\theta \partial A \partial \Omega}$	$\mathrm{W \cdot cm^{-2} \cdot sr^{-1}}$
Wavelength	λ		micron (μ)
Spectral radiant power	P_λ	$P_\lambda = \frac{\partial P}{\partial \lambda}$	$\mathrm{W \cdot \mu^{-1}}$
Spectral radiant intensity	J_λ	$J_\lambda = \frac{\partial J}{\partial \lambda}$	$\mathrm{W \cdot sr^{-1} \cdot \mu^{-1}}$
Spectral radiant emittance	W_λ	$W_\lambda = \frac{\partial W}{\partial \lambda}$	$\mathrm{W \cdot cm^{-2} \cdot \mu^{-1}}$
Spectral irradiance	H_λ	$H_\lambda = \frac{\partial H}{\partial \lambda}$	$\mathrm{W \cdot cm^{-2} \cdot \mu^{-1}}$
Spectral radiance	N_λ	$N_\lambda = \frac{\partial N}{\partial \lambda}$	$\mathrm{W \cdot cm^{-2} \cdot sr^{-1} \cdot \mu^{-1}}$
Radiant emissivity	ϵ	Ratio of "emitted" radiant power to that from an ideal blackbody at the same temperature.	
Radiant absorptance	α	Ratio of "absorbed" radiant power to incident radiant power.	
Radiant reflectance	ρ	Ratio of "reflected" radiant power to incident radiant power.	
Radiant transmittance	τ	Ratio of "transmitted" radiant power to incident radiant power.	

Note:

The spectral radiant emissivity $\epsilon(\lambda) \equiv W_\lambda / W_{\lambda,BB} \neq \partial\epsilon/\partial\lambda$. Hence, the subscript notation ϵ_λ, which could be confused with $\partial\epsilon/\partial\lambda$, is not recommended, although it is often used. Similarly, it is recommended that the spectral absorptance, spectral reflectance, and spectral transmittance be written as $\alpha(\lambda)$, $\rho(\lambda)$, and $\tau(\lambda)$, respectively.

* Reproduced from Reference 1, based on References 2 and 3.

In Table I, the definitions given for radiant em. ssivity and radiant reflectance take no account of the effects of the geometry of the radiation beam. The following treatment will refine these definitions to recognize explicitly the way in which these quantities may vary with orientation (relative to the surface). Only opaque surfaces (of zero transmittance) and the geometrical ray optics of incoherent radiation will be considered.

3. DIRECTIONAL REFLECTANCE

Consider a radiation field, where the radiance N_i is a function of both position and direction, incident on the surface of an opaque body where some of the radiation is absorbed and the rest is reflected (as used here, "reflected" includes diffuse reflectance or scattering) to form a second radiation field, where the radiance N_r of the reflected radiation is also a function of position and direction. N_r is directly proportional to N_i in the sense that, if the value of N_i is multiplied by a constant that is independent of position and direction, the resulting values of N_r will all be multiplied by the same constant factor. It will be seen below that the interdependence of the spatial and directional distributions of N_r and N_i is more complex.

Next, consider only the radiant power incident on a particular element δA of a reflecting surface through an elementary beam of solid angle $\delta\Omega_i$ from a direction (θ_i, φ_i), where θ_i is the angle from the normal to δA and φ_i is the azimuth about that normal (see Figure 1). This incident radiant power is given by [1]

$$\delta P_i(\theta_i, \varphi_i) = N_i(\theta_i, \varphi_i) \cos\theta_i \, \delta A \, \delta\Omega_i$$

$$= N_i(\theta_i, \varphi_i) \, \delta\Omega'_i \, \delta A \qquad [w], \qquad (1)$$

where $\delta\Omega'_i = \cos\theta_i \, \delta\Omega$

$$= \sin\theta_i \cos\theta_i d\theta_i d\varphi_i$$

is the "projected solid angle" [1, 4] of the elementary beam. Correspondingly, the irradiance at δA is

$$\delta H_i(\theta_i, \varphi_i) = N_i(\theta_i, \varphi_i) \, \delta\Omega'_i \qquad [w \cdot cm^{-2}]. \qquad (2)$$

Then the radiant intensity of the surface element δA, due to reflection (scattering) of radiation from this incident elementary beam, in the direction (θ_r, φ_r) is

$$\delta J_r(\theta_r, \varphi_r) = \rho'(\theta_i, \varphi_i, \theta_r, \varphi_r) \cos\theta_r \, \delta P_i(\theta_i, \varphi_i) \qquad [w \cdot sr^{-1}] \qquad (3)$$

or, by dividing both sides of Equation (3) by $\delta A \cos\theta_r$, we obtain the reflected (scattered) radiance

$$\delta N_r(\theta_r, \varphi_r) = \rho'(\theta_i, \varphi_i, \theta_r, \varphi_r) \, \delta H_i(\theta_i, \varphi_i) \qquad [w \cdot cm^{-2} \cdot sr^{-1}], \qquad (4)$$

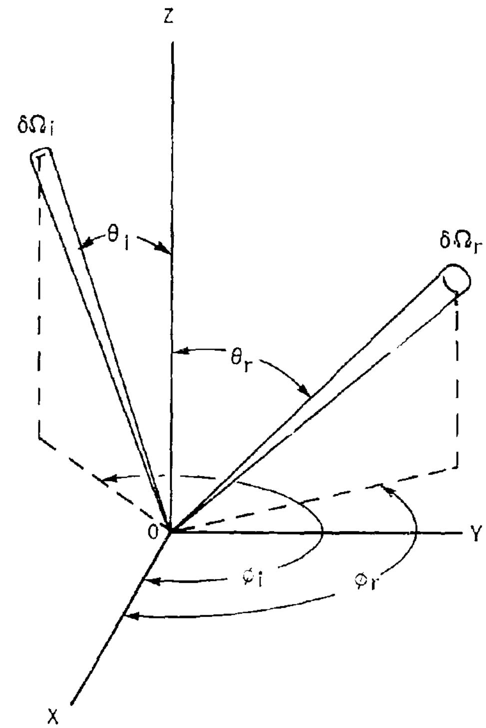

Figure 1 Geometry of incident and reflected elementary beams. (Z-axis is chosen along the normal to the surface element at O.)

where $\rho'(\theta_i, \varphi_i, \theta_r, \varphi_r) \quad \dfrac{\delta N_r(\theta_r, \varphi_r)}{\delta H_i(\theta_i, \varphi_i)} = \dfrac{\delta N_r(\theta_r, \varphi_r)}{N_i(\theta_i, \varphi_i)\, \delta \Omega'_i} \quad [sr^{-1}]$ (5)

is the partial reflectance or "reflection-distribution function"[5] of the surface element δA for radiation incident from the direction (θ_i, φ_i) and reflected (scattered) in the direction (θ_r, φ_r). Furthermore, by a reciprocity theorem of wide generality[6, 7] first enunciated by Helmholtz,* we may write

$$\rho'(\theta_1, \varphi_1, \theta_2, \varphi_2) = \rho'(\theta_2, \varphi_2, \theta_1, \varphi_1) \qquad [sr^{-1}]. \tag{6}$$

Thus $\rho'(\theta_1, \varphi_1, \theta_2, \varphi_2)$ is ordinarily the partial reflectance between the two directions (θ_1, φ_1) and (θ_2, φ_2), where either direction may be that of the incident elementary beam and the other that of the reflected (scattered) elementary beam.

Hence, we can write the expression for the radiance at a point of the reflecting surface (taken as the origin for spherical coordinates) in the direction (θ_r, φ_r) due to reflection (scattering) of all beams of incident radiation as

---

* A search for a proof (in English) of this important theorem also turned up a number of authors who referred to, or made use of, the theorem in various ways without giving a proof[8, 9, 10, 11, 12] including Von Helmholtz[13], although Planck[14] states, without specific citation, that Von Helmholtz "proved" the theorem. DeHoop[7] not only gives a proof (essentially the same as that of Kerr[6]) but also includes an explicit statement of the requisite conditions.

$$N_r(\theta_r, \varphi_r) \equiv \int_0^{2\pi} \int_0^{\pi/2} \rho'(\theta_i, \varphi_i, \theta_r, \varphi_r)\, N_i(\theta_i, \varphi_i) \sin\theta_i \cos\theta_i\, d\theta_i\, d\varphi_i$$

$$= \int_h \rho'(\theta_i, \varphi_i, \theta_r, \varphi_r)\, N_i\,(\theta_i, \varphi_i)\, d\,\Omega'_i \qquad [w \cdot cm^{-2} \cdot sr^{-1}]\ , \qquad (7)$$

where we adopt the following notation to designate integration over a hemisphere:

$$\int_h f\,(\theta, \varphi)\, d\Omega \equiv \int_0^{2\pi} \int_0^{\pi/2} f\,(\theta, \varphi) \sin\theta\, d\,\theta\, d\,\varphi$$

and $$\int_h f\,(\theta, \varphi)\, d\Omega' = \int_0^{2\pi} \int_0^{\pi/2} f\,(\theta, \varphi) \sin\theta \cos\theta\, d\,\theta\, d\,\varphi.$$

This relation -- Equation (7) -- is for a particular point, or for the surface element δA at that point. For a more general expression, we must also establish the reflected radiance from other points. When ρ' and N_i are expressed as functions of spatial location (as well as direction) for all points on the reflecting surface, Equation (7) gives the reflected radiance N_r as a function of position for these points on the reflecting surface, as well as for direction (θ_r, φ_r) at each such point. However, it is important in that case to recognize that Equation (7) is written above in coordinates which, for convenience, are specially oriented with respect to the surface element δA. Appropriate adjustments must be made when dealing with irregular surfaces where the direction of the normal, with respect to fixed coordinates, changes in going from one surface element to another.

Whether surface irregularities are treated as microscopic, (in the sense that their effects are integrated or averaged in the distribution function or partial reflectance $\rho'(\theta_1, \varphi_1, \theta_2, \varphi_2)$), or as macroscopic (in the sense that they may be analyzed into smaller surface elements δA for treatment as in the preceding paragraph) can be arbitrary, depending on the degree of resolution desired, or can be dependent on circumstances limiting achievable resolution. For example, in examining the reflectance of a highly irregular surface containing deep cavities, such as a piece of volcanic scoria, or a coarse, blackened cellulose sponge in the laboratory, it may be possible to consider the reflectance of different portions of the walls of single cavities (which are ther regarded as macroscopic irregularities). But when studying the possible effects of similar surfaces which may exist on the moon, where such fine detail cannot possibly be resolved by the best telescopes on earth, these are necessarily treated as microscopic irregularities.[15, 16] Still more complicated considerations are introduced when microscopic irregularities are small enough to have dimensions of the order of, or less than, the wavelength of the incident light or other electromagnetic radiation.[17, 18, 19, 20]

The total reflectance ρ of a surface element δA is defined in general as

$$\rho \equiv \delta P_r / \delta P_i \qquad [\text{dimensionless}], \qquad (8)$$

where δP_i is the total radiant power incident (from all directions) on δA, and δP_r is the total resulting reflected radiant power (in all directions). As stated above, the value of ρ depends upon the geometry and spectrum of the incident beam of radiation, which may be different in each particular case. Here for the moment we are concerned primarily with the geometrical relations. Hence, for the remainder of this paper, except where otherwise stated, we will eliminate spectral considerations by restricting the spectrum of the incident radiation to a region over which ρ does not change significantly with wavelength. It is then useful to consider some special cases of incident-beam geometry.

If the incident radiation is well collimated, within a small element of solid angle $\delta\Omega_i = \sin\theta_i d\theta_i d\varphi_i$ from the direction (θ_i, φ_i) the total radiant power incident on δA is

$$\delta P_i = \delta H_i (\theta_i, \varphi_i)\, \delta A \qquad [\mathrm{w}]. \tag{9}$$

Then, from Equation (5),

$$\begin{aligned} \delta N_r (\theta_r, \varphi_r) &= \rho' (\theta_i, \varphi_i, \theta_r, \varphi_r)\, \delta H_i (\theta_i, \varphi_i) \\ &= \rho' (\theta_i, \varphi_i, \theta_r, \varphi_r)\, \delta P_i / \delta A \qquad [\mathrm{w \cdot cm^{-2} \cdot sr^{-1}}] \end{aligned} \tag{10}$$

But
$$\delta P_r = \delta A \int_h \delta N_r (\theta_r, \varphi_r)\, d\Omega'_r$$
$$= \delta P_i \int_h \rho' (\theta_i, \varphi_i, \theta_r, \varphi_r)\, d\Omega'_r$$
$$= \delta P_i\, \rho_{di} (\theta_i, \varphi_i) \qquad [w], \qquad (11)$$

where $\rho_{di} (\theta_i, \varphi_i)$ is the (total) directional reflectance, for a well-collimated incident beam, given by

$$\rho_{di} (\theta_i, \varphi_i) \equiv \int_h \rho' (\theta_i, \varphi_i, \theta_r, \varphi_r)\, d\Omega'_r \qquad [\text{dimensionless}]. \qquad (12)$$

For isotropic surfaces, there is no dependence on the azimuth φ and Equation (12) simplifies to the frequently recognized dependence on θ: $\rho_{di} (\theta_i, \varphi_i) = \rho_{di} (\theta_i)$. If the well-collimated beam is incident perpendicularly on a plane surface, we have the commonly-reported normal reflectance, $\rho_n = \rho_{di} (0)$. If a point on the surface of a solid is uniformly irradiated from all external directions, i.e., if N_i is a constant, the reflected radiance in the direction (θ_r, φ_r), from Equation (7) is given by

$$N_r(\theta_r, \varphi_r) = N_i \int_h \rho' (\theta_i, \varphi_i, \theta_r, \varphi_r)\, d\Omega'_i$$
$$= \rho_{dr} (\theta_r, \varphi_r)\, N_i \qquad [w \cdot cm^{-2} \cdot sr^{-1}]; \qquad (13)$$

where
$$\rho_{dr} (\theta_r, \varphi_r) \equiv \int_h \rho' (\theta_i, \varphi_i, \theta_r, \varphi_r)\, d\Omega'_r \qquad [\text{dimensionless}]. \qquad (14)$$

But, from the reciprocity relation, Equation (6), and Equations (12) and (14) we can write

$$\rho_{di} (\theta_1, \varphi_1) = \rho_{dr} (\theta_1, \varphi_1) = \rho_d (\theta_1, \varphi_1) \qquad [\text{dimensionless}]. \qquad (15)$$

Thus, the (total) directional reflectance $\rho_d(\theta_1, \varphi_1)$ for a well-collimated beam incident from the direction (θ_1, φ_1) is also the ratio between the reflected radiance $N_r(\theta_1, \varphi_1)$ in that same direction and the incident radiance N_i when the surface is uniformly irradiated from all directions (hemispherical irradiation). This relation -- Equations (13) and (15) -- is the basis for a reflectometry technique described by McNicholas.[8]

4. DIRECTIONAL EMISSIVITY (AND ABSORPTANCE)

More important, Equations (11), (13) and (15) are the basis for evaluating and equating the directional absorptance and directional emissivity of the surface element δA in a simple relation which has the same form as the Kirchhoff's-Law relation --see Equation (18) below. If, in Equation (13), the uniform incident radiance N_i is equal to $N_b(T)$, the blackbody radiance (either total or spectral, i.e., in a small wavelength interval at a given wavelength) in an isothermal enclosure at T^oK, and if, in fact, the reflecting surface forms the wall of such an enclosure so that it, too, is at this same temperature, then the radiance in the direction (θ_1, φ_1) from the element of wall surface δA is made up of an emitted radiance and a reflected radiance, as follows:

$$N_e + N_r = \epsilon_d(\theta_1, \varphi_1) N_b(T) + \rho_{dr}(\theta_1, \varphi_1) N_b(T)$$

$$= N_b(T) \qquad [w \cdot cm^{-2} \cdot sr^{-1}]. \qquad (16a)$$

Similarly, of the radiance $N_b(T)$ incident on the element δA from a direction (θ_1, φ_1), a portion N_a is absorbed and the remainder N_{ir} is reflected (scattered) in all directions (into a hemisphere):

$$N_a + N_{ir} = \alpha_d (\theta_1, \varphi_1) N_b (T) + \rho_{di} (\theta_1, \varphi_1) N_b (T)$$
$$= N_b (T) \qquad [w \cdot cm^{-2} \cdot sr^{-1} \qquad (16b)$$

Here, $\epsilon_d (\theta_1, \varphi_1)$ is the directional emissivity (at temperature T) of the element δA for radiation emitted in the direction (θ_1, φ_1) and $\alpha_d (\theta_1, \varphi_1)$ is the absorptance (at T) for radiation incident from that direction. Consequently, from Equation (15),

$$\epsilon_d (\theta_1, \varphi_1) = 1 - \rho_{dr} (\theta_1, \varphi_1)$$
$$= 1 - \rho_{di} (\theta_1, \varphi_1) = \alpha_d (\theta_1, \varphi_1) \qquad [\text{dimensionless}]. \quad (17)$$

Note that equilibrium maintenance with conservation of energy (Kirchhoff's Law) by itself would justify only each line of Equation (17) independently, and the Helmhotz Reciprocity Law (which is the basis for Equation (6) and, in turn, Equation (15)) must also be invoked in order to equate them to each other and so to relate emissivity for radiation emitted into a given direction to the absorptance for radiation incident from that same direction (See Appendix A concerning a contrary position.)

In the more familiar form of Kirchhoff's Law,

$$\epsilon = 1 - \rho = \alpha \qquad [\text{dimensionless}], \qquad (18)$$

directional quantities are not considered. Instead, the total emissivity for radiation emitted in all directions (into a hemisphere) is related to the total reflectance (in all directions into a hemisphere) for uniform incident radiance (from all directions, i.e., from a hemisphere) and to the total absorptance for uniform incident radiance (from all directions, i.e., from a hemisphere). The total reflectance ρ in Equation (18), for uniform incident radiance (N_i = a constant independent of direction) is then

$$\rho = \delta P_r / \delta P_i = \frac{\delta A \int_h N_r \, \delta\Omega'}{\delta A \int_h N_i \, \delta\Omega'} = \frac{N_i \int_h \rho_d(\theta, \varphi) \, d\Omega'}{N_i \int_h d\Omega'}$$

$$= \frac{1}{\pi} \int_h \rho_d \, (\theta, \varphi) \, d\Omega' \qquad [\text{dimensionless}]. \qquad (19)$$

The quantities in Equation (18) are those involved in heat-transfer computations where the interest is in the net flow of energy across a bounding surface, involving radiation received, emitted, or reflected in all directions.

Equations (17) and (18) apply in all cases to spectral radiation (i.e., the radiation in a very small wavelength interval about a specified wavelength) and hence also to any spectral interval in which ρ or ρ_d (and therefore also ϵ or ϵ_d and α or α_d) do not change significantly with wavelength. When thermal equilibrium exists (i.e., when $N_i = N_b\,(T) = \int_0^\infty N_{\lambda b}(T, \lambda)\, d\lambda$, where $N_{\lambda b}\,(T, \lambda)$ is the spectral radiance of a blackbody at $T\,^{\circ}K$), they also apply to total radiation (all wavelengths), even though the spectral reflectance varies with wavelength. However, if the spectral reflectance is not a constant and the spectral distribution of the incident radiation is arbitrary (non-equilibrium condition), Equations (17) and (18) do not necessarily hold for the total (all wavelengths) reflectance, absorptance, and emissivity.

EXAMPLES.

In order to clarify the foregoing treatment of reflectance, it may be helpful to apply it to some frequently encountered situations. In reflectance measurements, it is a common practice to make comparisons with standard surfaces which approximate the limiting cases of perfectly diffuse reflectance (MgO is often used) and specular reflectance (a highly polished mirror).

First, a perfectly diffuse reflector is characterized by a constant value of partial reflectance ρ' in all directions. If such a surface is diffusely irradiated (N_i constant over a hemisphere) and the reflected radiation in a well-collimated beam in any particular direction is measured, or, in the reverse situation, if well-collimated incident radiation is reflected into a hemispherical receiver (e.g., an integrating sphere), the ratio of reflected power (flux) from a given surface area to the incident power on that area is given in either case by the directional reflectance ρ_d which, by Equation (12), is also a constant:

$$\rho_d = \rho' \int_h d\Omega' = \pi \rho' \qquad \text{[dimensionless]}. \tag{20}$$

Hence, from Equation (13), the total reflectance for any arbitrary configuration of incident radiation is given by

$$\rho = \frac{P_r}{P_i} = \frac{\int\int_h N_r \, d\Omega' dA}{\int\int_h N_i \, d\Omega' dA} = \frac{\rho_d \int\int_h N_i \, d\Omega' dA}{\int\int_h N_i \, d\Omega' dA}$$

$$= \rho_d = \pi\rho' \qquad \text{[dimensionless]}, \qquad (21)$$

where the integration with respect to dA is carried out over the same area in both numerator and denominator, and N_i may be any function of direction and position. When the incident radiation is uniformly distributed over the surface (even though it is not necessarily uniform with respect to incident direction), the reflected radiance N_r in any direction is related to the irradiance H_i by the partial reflectance ρ' defined in Equation (5), as follows:

$$\rho' = N_r/H_i = \rho/\pi \qquad [\text{sr}^{-1}]. \qquad (22)$$

Second, a perfectly specular reflec... ...haracterized by the relation

$$N_r(\theta, \varphi \pm \pi) = \rho_d\,(\theta, \varphi)\; N_i\,(\theta, \varphi) \qquad [\text{w} \cdot \ldots \cdot \text{sr}^{-1}]. \qquad (23)$$

By comparing this with the general relationship between incident and reflected radiances, it can be seen that Equation (23) will result if the partial reflectance ρ' in Equation (7) has the form

$$\rho'\,(\theta_i, \varphi_i, \theta_r, \varphi_r) = 2\,\rho_d\,(\theta_i, \varphi_i)\;\delta(\sin^2\theta_r - \sin^2\theta_i)\;\delta(\varphi_r - \varphi_i \pm \pi) \qquad [\text{sr}^{-1}] \qquad (24)$$

where $\delta\,(\sin^2\theta_r - \sin^2\theta_i)$ and $\delta(\varphi_r - \varphi_i \pm\pi)$ are Dirac delta-functions which satisfy the defining relations

δ (u) = 0 for u $\neq$ 0,

$\int \delta$ (u) du = 1, and

$\int$ f (u) δ (u) du = f (0),

when the integration is carried out over the full range of the variable, $0 \leq \theta \leq \pi/2$ and $0 \leq \varphi \leq 2\pi$, in each case.

Sometimes attempts are made to state apparently simple relationships between the output of a given reflectometer for a diffuse standard surface and a specular standard surface. This is not a simple matter. It depends critically upon the configuration, and a wide variety of configurations are employed.[21]

As an illustration, assume that a sample surface is uniformly irradiated by a well-collimated beam of uniform radiance N_i within a small solid angle $\Delta\Omega_i$ incident from the direction (θ, φ). Assume also that a detector is placed with appropriate optics (stops and, if necessary, focussing elements) to insure that it receives radiation only from a well-defined portion of the irradiated surface, of area Δ A, through a well-defined solid angle, $\Delta\Omega_r < \Delta\Omega_i$, in the direction $(\theta, \varphi \pm \pi)$. First, if the reflecting surface is perfectly diffusing, the reflected radiance is constant in all directions and is related to the incident irradiance, $H_i = N_i \Delta \Omega_i \cos \theta_i = N_i \Delta \Omega'_i$, by Equations (5), and (21). The total power (flux) received by the detector is then

$$P_d = \int \int N_r \, d\Omega'_r \, dA$$

$$= (\rho/\pi) N_i \Delta \Omega'_i \int \int \cos \theta_r, d\Omega_r, dA$$

$$= (\rho/\pi) N_i \Delta\Omega'_i \Delta\Omega'_r \Delta A \qquad (25)$$

Note that if there is vignetting, so that the solid angle $\Delta\,\Omega_r$ through which the detector receives radiation is not exactly the same for each point of the surface Δ A, it may be difficult to evaluate the integrals.

Next, if a specular standard surface is substituted for the diffuse surface (and if it is carefully aligned to insure that the solid angle $\Delta\,\Omega_r$ is completely filled with reflected radiation), then, from Equation (23), the total power (flux) received by the detector can be written as

$$
\begin{aligned}
P_s &= \int\int N_r \, d\Omega' \, dA \\
&= \rho_d\,(\theta, \varphi)\, N_i \,\Delta\,\Omega'_r \,\Delta A \qquad [\mathrm{w}]\,. \qquad (26)
\end{aligned}
$$

If these were ideal standards, with reflectance values of unity in each case ($\rho = \rho_d = 1$), the ratio of the detector outputs (proportional to received power) for the two surfaces under the described conditions would then be

$$
\frac{P_s}{P_d} = \frac{\pi}{\Delta\,\Omega'_i} = \frac{\pi}{\Delta\,\Omega_i \cos\,\theta_i} \qquad [\text{dimensionless}]\,. \qquad (27)
$$

It is obvious that this relation depends directly (inversely) on the solid-angle spread of the incident collimated beam. The dependence on the other factors in the configuration -- the alignment, the solid angle of acceptance of the detector, etc. -- is clear from the foregoing discussion and specification of the conditions for which this relation was derived.

Only what might be termed the external radiometric relations have been considered in the foregoing treatment and no attempt has been made to deal with the deeper theory relating reflectance, emissivity, and absorptance to the optical constants of the materials. A good summary of the most important aspects of that approach is given in Reference 22.

C. SUMMARY.

The partial reflectance of a surface element $\rho'(\theta_1, \varphi_1, \theta_2, \varphi_2)$ is defined in Equation (5) as the ratio between the reflected radiance in the direction (θ_2, φ_2) and the incident irradiance from the direction (θ_1, φ_1) which produces it. Integration of this quantity over the solid angle of a hemisphere in Equation(12) yields the directional reflectance $\rho_d(\theta_1, \varphi_1)$, which is the fraction of the radiant power incident from the direction (θ_1, φ_1) that is reflected in all directions (into a hemisphere). Furthermore, if the reciprocity theorem--Equation (6) -- is applicable, as it ordinarily is, at least to a good approximation, then this directional reflectance is also the ratio between the radiance in the given direction and the incident radiance when the surface element is uniformly irradiated from all directions, as indicated in Equation (15).

The emissivity of an opaque surface element in the direction (θ_1, φ_1) is related to the directional reflectance $\rho_{dr}(\theta_1, \varphi_1)$ as shown in Equation (17). When the reciprocity theorem is applicable (as is usually the case), the emissivity in a given direction is equal to the absorptivity for radiation incident from that direction, which is also equal to one minus the directional reflectance for that same direction.

A perfectly diffuse reflector is characterized by uniform reflectance in all directions. It is shown in Equation (21) that this is equal to pi times the partial reflectance. The relationship between the partial and directional reflectances for a perfectly specular reflector involves Dirac delta-functions, as given in Equation (24).

The completely general expression, relating the reflected radiance of a surface element in a given direction $N_r(\theta_r, \varphi_r)$ to the incident field of radiation, specified by expressing the incident radiance as a function of direction $N_i(\theta_i, \varphi_i)$, is given in terms of the partial reflectance $\rho'(\theta_i, \varphi_i, \theta_r, \varphi_r)$ by Equation (7). This holds true regardless of the geometrical configuration of the incident beam. The radiant power in a reflected beam is then computed by integrating the resulting value of reflected radiance, as a function of direction, over the appropriate projected area and solid angle as indicated in the first line of Equation (25) and discussed in greater detail in Reference 1.

As with many idealized physical quantities, partial and directional reflectances can never be measured exactly, even with perfect instrumentation. Since a measurement requires a beam of radiation of non-zero cross section and solid angle, the measurement at best can only yield average values over these intervals of projected area and solid angle. However, the concepts, terminology, and symbols presented here make it possible to specify explicitly and unambiguously the interrelationships and approximations involved in dealing with real situations. Also, the application of Kirchhoff's Law -- Equation (18) -- to the directional quantities can be stated explicitly, as in Equation (17).

7 APPENDIX A -- Reciprocity in an Isothermal Enclosure

A paper by Bauer[23] presents an alleged proof of the Helmholtz reciprocity law for diffuse reflection as a consequence of equilibrium conditions in an isothermal enclosure. The argument hinges on the statement that the second law of thermodynamics requires that there be no net exchange of energy by radiation between any two individual elements of the internal (opaque) surface of an isothermal enclosure. However, it seems to me that the requirements of the second law apply to the total flow of energy, taking into account radiation emitted into, and absorbed from, all directions (full hemisphere) by such a surface element, as the basis for Kirchhoff's Law (Equation 18).

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Electronic Defense Labs., Mountain View, Calif.
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Report No. EDL-G266, 19 May 1964

Concepts, terminology, and symbols are presented for specifying and relating directional variations in reflectance and emissivity of an opaque surface element. Their relationship to more familiar concepts, including those of perfectly diffuse and specular reflectance is given, and they are applied to illustrative examples. It is shown that, when the usual reciprocity relationship holds, the reflectance for a ray incident on an opaque surface element is related by Kirchhoff's Law to the emissivity of that element for a ray emitted along the same line in the opposite sense.

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3. Emissivity
4. Opaque
5. Surfaces
6. Variation
7. Ray
8. Incidence
9. Kirchhoff's
10. Law
11. Radiance
12. Microscopic
13. Irregularities
14. Reflected
15. Power
16. Diffuse
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