Sfb 1946 A1

Coupled Simulation of Heterogeneous Hydrological Systems: Numerical Modeling of Runoff Generation in Lowland Areas

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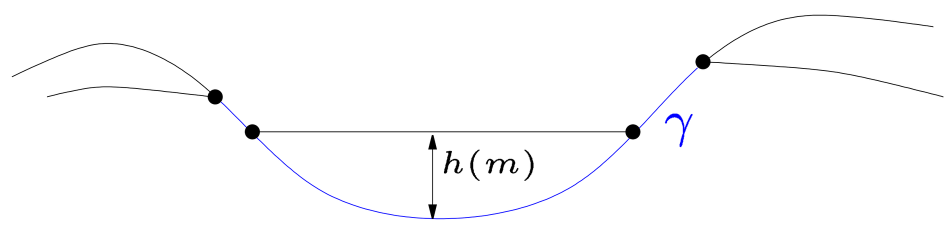
Coupled Model for Ground and Surface Water



- Signorini-type problem for Richards equation

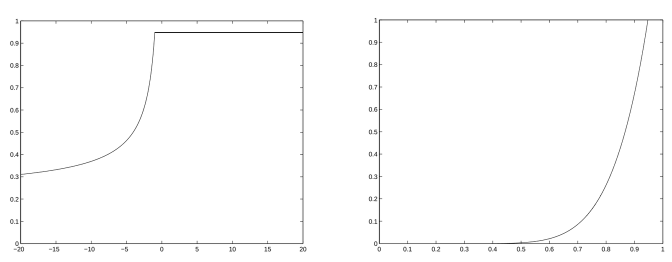
$$\frac{\partial}{\partial t}\theta(p) + \operatorname{div} \mathbf{v}(x,p) = 0, \quad \mathbf{v}(x,p) = -\frac{K(x)}{\mu}\kappa\left(\theta(x,p)\right)\nabla(p - \rho g z)$$

- mass conservation of surface water

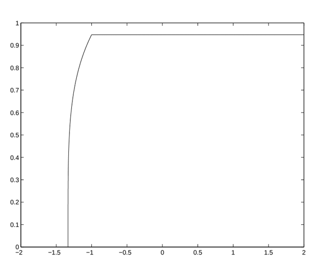
$$\dot{m} = \rho \int_{\gamma(t)} \mathbf{v}(x,p)\cdot \mathbf{n}\, d\sigma$$

Numerical Challenges

- nonlinear, heterogeneous state equations
- multiple dynamic free boundaries
- γ ill-conditioned by geometry
- dynamic coupling of ODEs via geometry
- anisotropic computational domain Ω
- strongly varying permeability $K(x)$



saturation $\theta(p)$ and rel. permeability $\kappa(\theta)$



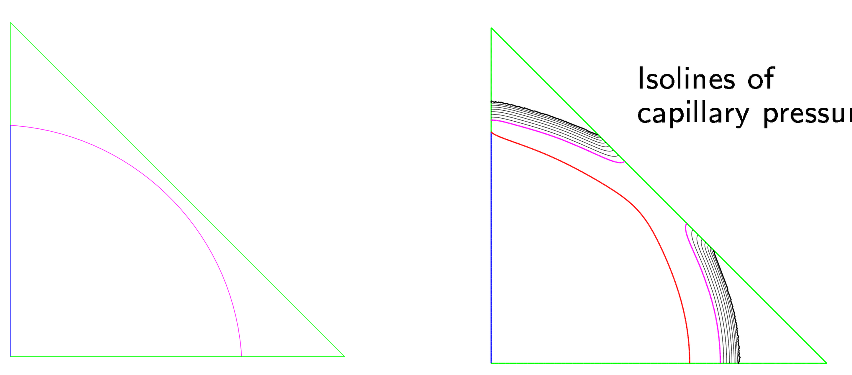
$\mathcal{H}(u) = \theta(\beta^{-1}(u))$
Kirchhoff transformation $u = \beta(p)$

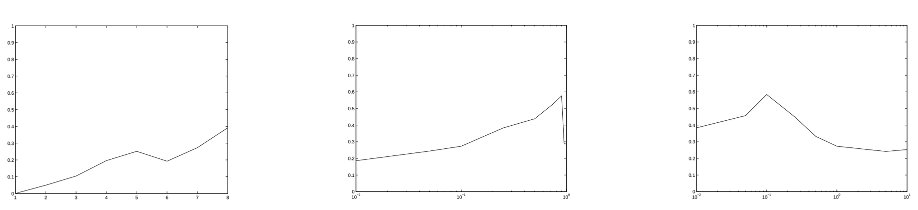
Monotone Multigrid (FU)

Relies on Kirchhoff transformation β and monotonicity of $\mathcal{H} = \theta(\beta^{-1}(\cdot))$. This includes Signorini-type boundary conditions and implies robustness.

Model problem with $\rho = 0$:

$\Delta t = 0.1 \hat{=}\ 20\ s$
$\lambda = 1$
$p_b = -0.1\ m$
$n = 0.33$
$K = 1.6 \cdot 10^{-3}\ m/s$

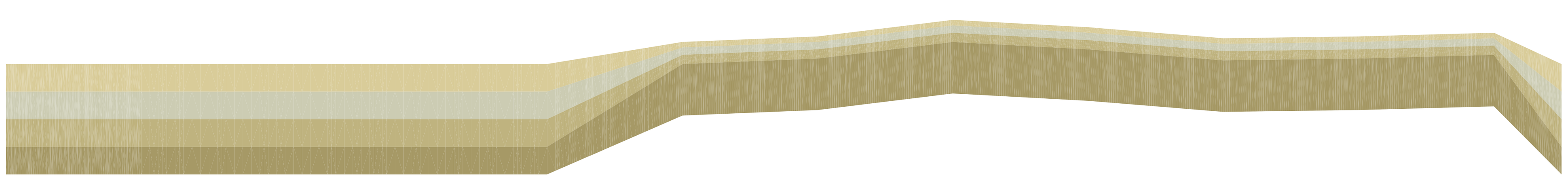




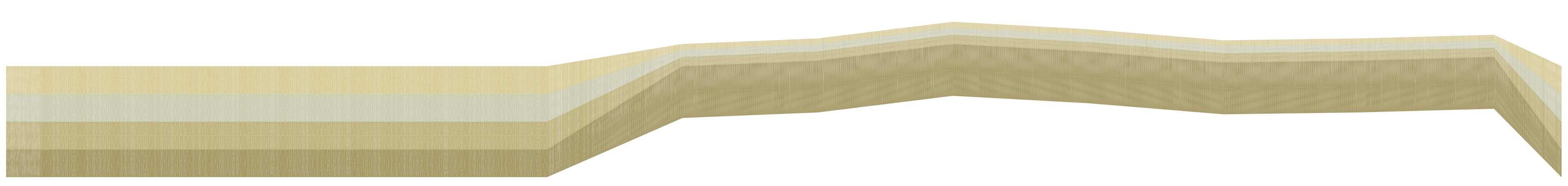
Robustness with respect to Δx, Δt and λ.

Algebraic Newton Multigrid (WIAS)

Directly applicable to heterogeneous state equations.



Initial condition: low ground water table (isolines of capillary pressure)



Risen ground water after rainfall (isolines of capillary pressure)

Solution of Richards equation by Newton method and AMG preconditionend BiCGstab solver: Influence of average rainfall on ground water table.

Goals and Research Strategy

ODE PART

Implicit adaptive time discretization of the compartment equations leads to a nonlinear algebraic system of the form

$$m^{k+1} = F(\mathbf{v}(m^{k+1})).$$

F incorporates geometry, e.g. via *merging of compartments*.
Each step of iterative solution requires evaluation of $\mathbf{v}(m^{k+1})$, i.e. solution of the PDE.

PDE PART

We aim at

robust and flexible multigrid solvers

for discretized spatial problems based on

- Algebraic Newton Multigrid,
- Monotone Multigrid (MMG).

Anisotropy is incorporated by

- vertical line smoothing.

Strongly varying permeability $K(x)$ is treated by

- algebraic multigrid techniques.

Reduction of unknowns and time steps is achieved by

- adaptive selection of time steps,
- adaptive local mesh refinement.

Implementation is based on

- the software platform PDELIB.

Work Plan

Cooperation within the Sfb:
- Standard numerical techniques for the ode part

Bänsch:
- Algebraic Newton multigrid
- Implementation of dynamic coupling with geometry in the framework of PDELIB
- A posteriori error indicators

Kornhuber:
- Assume homogeneous state equations
- Incorporation of gravity into MMG
- Line smoothing by 1D-MMG

Kornhuber, Bänsch:
- Computations of the fully coupled model
- Combination of Newton linearization and MMG for heterogeneous state equations

Kornhuber, Bänsch, Bronstert:
- Numerical assessment of fully coupled and decoupled models