Heiko Berninger

Freie Universität Berlin

http://www.math.fu-berlin.de/~berninge

On a Method of Robin–Type for the Richards Equation in Heterogeneous Soil

The Richards equation is a doubly nonlinear elliptic–parabolic pde describing saturated–unsaturated groundwater flow. We assume the porous medium to be heterogeneous, i.e. constituted by subdomains with different soil types. Then the nonlinearities depending on the soil parameters change discontinuously across the interfaces between these subdomains. In our time–discretization of the Richards equation, the gravitational part is taken explicitly. Then Kirchhoff transformation, applied separately in the subdomains, leads to local convex minimization problems. These local problems can be treated efficiently and robustly using monotone multigrid [2]. Nonlinear nonoverlapping domain decomposition like the Dirichlet–Neumann or the Robin method provides a coupling of these problems. Here we focus on the Robin method. We present a nonlinear Steklov–Poincaré theory for a method of Robin–type applied to the Richards equation and related problems. This leads us to a convergence result for the Richards equation in one space dimension. Finally, numerical results show the efficiency of the Robin method for our problem in higher dimensions.

[1] H. Berninger. *Domain Decomposition Methods for Problems with Jumping Nonlinearities and Application to the Richards Equation.* Dissertation, FU Berlin, in preparation.

[2] R. Kornhuber. On Constrained Newton Linearization and Multigrid for Variational Inequalities. *Numer. Math. 91* (2002), 699–721.