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On a Substructuring Method of Robin–Type for the Richards Equation

The Richards equation is a doubly nonlinear elliptic–parabolic pde describing saturated–unsaturated flow in porous media. We present a method by which the Richards equation can be solved applying monotone multigrid even if the nonlinearities depending on the soil parameters change discontinuously across certain subdomains. This is done by first using a time–discretization in which the gravitational part is taken explicitly. Then the obtained quasilinear elliptic equation is reduced to a semilinear equation by Kirchhoff transformation in the subdomains with homogeneous soil. These subproblems can be written as convex minimization problems that can now be treated efficiently and robustly using monotone multigrid [2]. The problems in the subdomains can be coupled using nonlinear nonoverlapping domain decomposition like the Dirichlet–Neumann or the Robin method. Here we concentrate on the Robin method. We present a nonlinear Steklov–Poincare theory for a method of Robin–type and we prove a convergence result for the Richards equation in one space dimension. Numerical results show the efficiency of the method also in higher dimensions.

Joint work with Ralf Kornhuber and Oliver Sander (Freie Universität Berlin).

[1] H. Berninger. *Nonlinear Domain Decomposition Methods for Saturated–Unsaturated Flow in Porous Media.* Dissertation, FU Berlin, in preparation.

[2] R. Kornhuber. On Constrained Newton Linearization and Multigrid for Variational Inequalities. *Numer. Math. 91* (2002), 699–721.