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Non-overlapping Domain Decomposition for the Richards Equation via Nemytskij Operators

We present new results on transmission problems related to the Richards equation in heterogeneous porous media. The Richards equation, which describes saturated-unsaturated groundwater flow, is discretized in time with an explicit treatment of the gravitational term. Then, different Kirchhoff transformations on the subdomains containing different soil-types lead to a coupling of local convex minimization problems across the interfaces. The nonlinear coupling is provided by Nemytskij operators acting on the trace space. The corresponding transmission conditions give rise to nonlinear Dirichlet–Neumann or Robin methods for which convergence results have been obtained in one space dimension ([1], [2]). We solve the local problems efficiently and robustly by monotone multigrid [3]. For the domain decomposition iterations, too, no further linearization is applied. Our numerical results provide a detailed comparison of the Dirichlet–Neumann method and the Robin method for problems related to the stationary Richards equation in 2D. Furthermore, we present a numerical example in 2D wherein we apply the Robin method to the Richards equation in four different soils with surface water and realistic hydrological data.

Joint work with R. Kornhuber and O. Sander, FU Berlin.

[1] H. Berninger. *Domain Decomposition Methods for Elliptic Problems with Jumping Nonlinearities and Application to the Richards Equation.* Dissertation, FU Berlin, October 2007.

[2] H. Berninger, R. Kornhuber and O. Sander. On nonlinear Dirichlet–Neumann algorithms for jumping nonlinearities. In: O.B. Widlund and D.E. Keyes, editors, *Domain Decomposition Methods in Science and Engineering XVI*, pp. 483–490, Springer, 2007.

[3] R. Kornhuber. On constrained Newton linearization and multigrid for variational inequalities. *Numer. Math.*, 91:699–721, 2002.