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Nonlinear Dirichlet–Neumann and Robin Methods for the Richards equation in heterogeneous soil

We present numerical solution techniques for highly nonlinear transmission problems which, for instance, may arise from the Richards equation in heterogeneous soil. The overall aim is to solve the problems without applying any linearization.

Our approach to the Richards equation is based on Kirchhoff transformation in case of a homogeneous porous medium. Here, convex minimization problems can be obtained for which monotone multigrid methods are robust solvers. With regard to the heterogeneous situation we assume that different homogeneous soils are located in different subdomains. As coupling conditions we impose continuity of the pressure and the water flux across the interfaces between the subdomains. Nonlinear Dirichlet–Neumann or, alternatively, Robin methods serve as iteration procedures for the coupled homogenous problems. Within our finite element discretization gravity is treated explicitly in time by an upwind technique which is obtained via an additive viscosity term. Finally, by a coupling of the Richards equation to surface water we demonstrate that our solver can also determine seepage faces around lakes.

In the talk we will explain our solution method and mention some analytical results in 1D. Furthermore, different numerical results in 2D shall be presented. Here, a detailed and surprising comparison of the Dirichlet–Neumann and the Robin method for certain test cases will be given. In addition, the applicability of our solution technique for the Richards equation shall be demonstrated in heterogeneous settings with hydrologically realistic soil parameters.

Joint work with: R. Kornhuber and O. Sander (FU Berlin)