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Minisymposium *"Modelling and Simulation of Coupled Surface and Groundwater Flow"*

Coupling of ground and surface water

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We present a numerical solution method for the coupling of the Richards equation in heterogeneous porous media with surface water. The latter can either be given as a compartment model with non-moving surface water (i.e. a lake) or as the depth-averaged shallow water equations.

Our approach to treat the Richards equation is based on the Kirchhoff transformation which can be carried out in case of homogeneous soil. As a result convex minimization problems are obtained to which monotone multigrid methods can be applied. We assume that different homogeneous soil-types are located in different subdomains while the coupling condition imposes continuity of the pressure and the water flux across the interfaces between the subdomains. This heterogeneity is addressed via a non-overlapping domain decomposition method, more concretely a nonlinear Robin method providing for an iteration of the coupled homogenous problems. Gravity is discretized explicitly in time using an upwind technique given by a viscosity term within our finite element discretization.

Both the free boundary of the seepage face around surface water coupled with the Richards equation and the free boundary separating the saturated from the unsaturated regime can be determined by our solver.

The coupling with surface water is provided by the hydrostatic pressure as a Dirichlet boundary condition for the Richards equation and the mass conservation which regulates the hight of the surface water by the water flux across the upper boundary. In case of the shallow water equations the water flux appears as a source term on the right hand side of the continuity equation. The shallow water equations are explicitly discretized by a Lax–Friedrichs scheme. Since we assume a smaller time step size for the shallow water equations than for the Richards equation we propose a semi-implicit discrete coupling which we treat by a fixed point iteration of Dirichlet–Neumann type.

In the talk we give a presentation of our solution method. Furthermore, we present numerical results which demonstrate that our method can be successfully applied to coupled ground and surface water problems with usual soil parameters in practical situations.

[1] H. Berninger, *Domain Decomposition Methods for Elliptic Problems with Jumping Nonlinearities and Application to the Richards Equation*, PhD thesis, Freie Universität Berlin, 2007