

Barbara MacCluer: Elementary Functional Analysis.

Books on Functional Analysis are not exactly ubiquitous, but every publishing house has several text books in this area on its backlist. Barbara MacCluer's book is an excellent new entry.

The book consists of six chapters. Chapter 1 introduces the fundamentals of Hilbert spaces like orthogonality, the projection theorem, the Fréchet-Riesz theorem and orthonormal bases. Chapter 2 deals with the elementary theory of bounded linear operators including their adjoints in the Hilbert and Banach space settings. The following chapter, called "The Big Three," provides proofs of the Hahn-Banach extension theorem, the uniform boundedness principle and the open mapping and closed graph theorems, and Chapter 4 introduces compact operators, mainly on Hilbert spaces, and their spectra. Here, the Fredholm alternative and the spectral theorem for compact self-adjoint operators are proved.

While the previous topics might be considered elementary, this can arguably not be said about the second half of the book (despite the book's title), covering the Gel'fand theory of commutative unital Banach algebras in Chapter 5 and the spectral theorem for bounded normal operators, in its multiplication operator form and its spectral measure form, in Chapter 6. Chapter 5 also contains a section on the weak* topology, in particular the Alaoglu theorem.

What distinguishes this book from many others is its reader-friendly style that is opposite to the dry definition-theorem-proof format found in many other expositions. Another noteworthy feature are the historical comments mingled into the text that help the reader understand the development of the subject. Each chapter is accompanied by a section of well-chosen, highly interesting and often non-canonical exercises (e.g., "What is wrong with the following argument?"). Since the author is an eminent operator theorist, Hardy and Bergman spaces and Toeplitz operators figure prominently among her examples.

Instructors should be aware, though, that various core results of Functional Analysis are only mentioned in passing in the text and are relegated to the exercises, like the duality of ℓ_p -spaces (not to mention the L_p -spaces), the Gram-Schmidt procedure or the fact that Banach spaces with compact unit ball are finite-dimensional. The notion of a reflexive space is barely spelt out, and the reflexivity of L_p not at all. Results with a real analysis flavour are generally quoted from other sources, e.g. that the trigonometric system is an orthonormal basis of L_2 . But the book contains a nice appendix on measure and integration.

In conclusion, this book presents a streamlined path to the spectral theorem for bounded operators. It is a pleasure to read and highly recommended.

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