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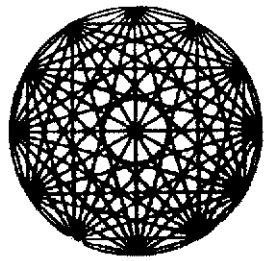

Computing

A Systolic Array Algorithm for the Algebraic Path Problem (Shortest Paths; Matrix Inversion)*

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Abstract — Zusammenfassung

A Systolic Array Algorithm for the Algebraic Path Problem (Shortest Paths; Matrix Inversion). It is shown how the Gauß-Jordan Elimination algorithm for the Algebraic Path Problem can be implemented on a hexagonal systolic array of a quadratic number of simple processors in linear time. Special instances of this general algorithm include parallelizations of the Warshall-Floyd Algorithm, which computes the shortest distances in a graph or the transitive closure of a relation, and of the Gauß-Jordan Elimination algorithm for computing the inverse of a real matrix.

AMS subject classifications: 68A05, (05C35, 05C38, 16A78, 65F05, 68E10).

CR categories and subject descriptors: C.1.2 [processor architectures]: multiple data stream architectures (multiprocessors) – systolic arrays; G.1.0 [numerical analysis]: general – parallel algorithms; G.1.3 [numerical analysis]: numerical linear algebra – matrix inversion; G.2.2 [discrete mathematics]: graph theory – path problems; B.6.1 [logic design]: design styles – cellular arrays; B.7.1 [integrated circuits]: types and design styles – algorithms implemented in hardware; VLSI (very large scale integration).

General terms: algorithms, design, performance.

Additional key words and phrases: Algebraic path problem, shortest paths, transitive closure, closed semirings, Gauß-Jordan elimination.

Ein systolic-array-Algorithmus für das algebraische Wegproblem (kürzeste Wege; Matrizeninversion). Es wird dargestellt, wie man den Gauß-Jordanschen Eliminationsalgorithmus für das algebraische Wegproblem auf einem hexagonalen systolischen Feld (systolic array) mit einer quadratischen Anzahl einfacher Prozessoren in linearer Zeit ausführen kann. Zu den Anwendungsbeispielen dieses allgemeinen Algorithmus gehört der Warshall-Floyd-Algorithmus zur Berechnung der kürzesten Wege in einem Graphen oder zur Bestimmung der transitiven Hülle einer Relation sowie der Gauß-Jordansche Eliminationsalgorithmus zur Inversion reeller Matrizen.

1. Introduction

The Algebraic Path Problem unifies three streams of evolution each of which independently developed its own algorithms: the determination of the transitive closure of a relation and the determination of shortest paths in networks; Kleene's

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